10th Mathematics

English Medium

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Learning Guide

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ONE MARK QUESTIONS
BOOKBACK

UNIT.I RELATIONS AND FUNCTIONS

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is
   (A) 1 (B) 2 **(C) 3** (D) 6

2. $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is
   (A) 8 (B) 20 **(C) 12** (D) 16

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true.
   (A) $(A \times C) \subset (B \times D)$ (B) $(B \times D) \subset (A \times C)$
   (C) $(A \times B) \subset (A \times D)$ (D) $(D \times A) \subset (B \times A)$

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is (A) 3 **(B) 2** (C) 4 (D) 8

5. The range of the relation $R = \{(x, x^2) \mid x \text{ is a prime number less than } 13\}$ is
   (A) $\{2, 3, 5, 7\}$ (B) $\{2, 3, 5, 7, 11\}$ **(C) $\{4, 9, 25, 49, 121\}$** (D) $\{1, 4, 9, 25, 49, 121\}$

6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is
   (A) $(2, -2)$ (B) $(5, 1)$ (C) $(2, 3)$ **(D) $(3, -2)$**

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is (A) m^n (B) n^m **(C) $2^{mn} - 1$** (D) 2^{mn}

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively
   (A) (8, 6) (B) (8, 8) (C) (6, 8) (D) (6, 6)

9. Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f: A \to B$ given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is a
   (A) Many-one function (B) Identity function
   (C) One-to-one function (D) Into function

10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is
    (A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ **(C) $\frac{2}{9x^2}$** (D) $\frac{1}{6x^2}$

11. If $f: A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to
    (A) 7 (B) 49 (C) 1 (D) 14

12. Let f and g be two functions given by $f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)\}$ $g = \{(0, 2), (1, 0), (2, 4), (-4, 2), (7, 0)\}$ then the range of $f \circ g$ is
    (A) $\{0, 2, 3, 4, 5\}$ (B) $\{-4, 1, 0, 2, 7\}$ (C) $\{1, 2, 3, 4, 5\}$ **(D) $\{0, 1, 2\}$**

13. Let $f(x) = \sqrt{1 + x^2}$ then (A) $f(xy) = f(x).f(y)$ (B) $f(xy) \geq f(x).f(y)$
    (C) $f(xy) \leq f(x).f(y)$ (D) None of these

14. If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are (A) $(-1, 2)$ **(B) $(2, -1)$** (C) $(-1, -2)$ (D) $(1, 2)$

15. $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is
(A) linear (B) cubic (C) reciprocal **(D) quadratic**

UNIT.II NUMBERS AND SEQUENCES

1. Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy
(A) $1 < r < b$ (B) $0 < r < b$ **(C) $0 \leq r < b$** (D) $0 < r \leq b$

2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are **(A) 0, 1, 8** (B) 1, 4, 8 (C) 0, 1, 3 (D) 1, 3, 5

3. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
(A) 4 **(B) 2** (C) 1 (D) 3

4. The sum of the exponents of the prime factors in the prime factorization of 1729 is
(A) 1 (B) 2 **(C) 3** (D) 4

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
(A) 2025 (B) 5220 (C) 5025 **(D) 2520**

6. $7^{4k} \equiv$ ________ (mod 100) **(A) 1** (B) 2 (C) 3 (D) 4

7. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is
(A) 3 (B) 5 (C) 8 **(D) 11**

8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P. (A) 4551 (B) 10091 **(C) 7881** (D) 13531

9. If 6 times of 6^{th} term of an A.P. is equal to 7 times the 7^{th} term, then the 13^{th} term of the A.P. is
(A) 0 (B) 6 (C) 7 (D) 13

10. An A.P. consists of 31 terms. If its 16^{th} term is m, then the sum of all the terms of this A.P. is
(A) $16\,m$ (B) $62\,m$ **(C) $31\,m$** (D) $\frac{31}{2}m$

11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?
(A) 6 (B) 7 **(C) 8** (D) 9

12. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \cdots + 2^0$ which of the following is true?
(A) B is 2^{64} more than A (B) A and B are equal
(C) B is larger than A by 1 **(D) A is larger than B by 1**

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \ldots$ is
(A) $\frac{1}{24}$ **(B) $\frac{1}{27}$** (C) $\frac{2}{3}$ (D) $\frac{1}{81}$

14. If the sequence $t_1, t_2, t_3, \ldots$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, \ldots$ is
(A) a Geometric Progression **(B) an Arithmetic Progression**
(C) neither an A.P. nor a G.P. (D) a constant sequence

15. The value of $(1^3 + 2^3 + 3^3 + \cdots + 15^3) - (1 + 2 + 3 + \ldots + 15)$ is
(A) 14400 (B) 14200 **(C) 14280** (D) 14520

UNIT.III ALGEBRA

1. A system of three linear equations in three variables is inconsistent if their planes
   (A) intersect only at a point (B) intersect in a line
   (C) coincides with each other **(D) do not intersect**

2. The solution of the system $x + y - 3z = -6,\ -7y + 7z = 7,\ 3z = 9$ is
   (A) $x = 1,\ y = 2,\ z = 3$ (B) $x = -1,\ y = 2,\ z = 3$
   (C) $x = -1,\ y = -2,\ z = 3$ (D) $x = 1,\ y = -2,\ z = 3$

3. If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
   (A) 3 **(B) 5** (C) 6 (D) 8

4. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is **(A) $\frac{9y}{7}$** (B) $\frac{9y^3}{(21y-21)}$ (C) $\frac{21y^2 - 42y + 21}{3y^3}$ (D) $\frac{7(y^2 - 2y + 1)}{y^2}$

5. $y^2 + \frac{1}{y^2}$ is not equal to (A) $\frac{y^4+1}{y^2}$ **(B) $\left[y + \frac{1}{y}\right]^2$** (C) $\left[y - \frac{1}{y}\right]^2 + 2$ (D) $\left[y + \frac{1}{y}\right]^2 - 2$

6. $\frac{x}{x^2 - 25} - \frac{8}{x^2 + 6x + 5}$ gives (A) $\frac{x^2 - 7x + 40}{(x-5)(x+5)}$ (B) $\frac{x^2 + 7x + 40}{(x-5)(x+5)(x+1)}$
   (C) $\frac{x^2 - 7x + 40}{(x^2 - 25)(x+1)}$ (D) $\frac{x^2 + 10}{(x^2 - 25)(x+1)}$

7. The square root of $\frac{256\, x^8\, y^4\, z^{10}}{25\, x^6\, y^6\, z^6}$ is equal to
   (A) $\frac{16}{5}\left|\frac{x^2 z^4}{y^2}\right|$ (B) $16\left|\frac{y^2}{x^2 z^4}\right|$ (C) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ **(D) $\frac{16}{5}\left|\frac{xz^2}{y}\right|$**

8. Which of the following should be added to make $x^4 + 64$ a perfect square
   (A) $4x^2$ **(B) $16x^2$** (C) $8x^2$ (D) $-8x^2$

9. The solution of $(2x - 1)^2 = 9$ is equal to
   (A) -1 (B) 2 **(C) $-1, 2$** (D) None of these

10. The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are
    (A) 100, 120 (B) 10, 12 **(C) $-120, 100$** (D) 12, 10

11. If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in __________
    (A) $A.P$ **(B) $G.P$** (C) Both $A.P$ and $G.P$ (D) none of these

12. Graph of a linear polynomial is a
    (A) straight line (B) circle (C) parabola (D) hyperbola

13. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is
    (A) 0 **(B) 1** (C) 0 or 1 (D) 2

14. For the given matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{bmatrix}$ the order of the matrix A^T is
    (A) 2×3 (B) 3×2 (C) 3×4 **(D) 4×3**

15. If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have
    (A) 3 **(B) 4** (C) 2 (D) 5

16. If number of columns and rows are not equal in a matrix then it is said to be a
(A) diagonal matrix **(B) rectangular matrix**
(C) square matrix (D) identity matrix

17. Transpose of a column matrix is
(A) unit matrix (B) diagonal matrix (C) column matrix **(D) row matrix**

18. Find the matrix X if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$
(A) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ **(B)** $\begin{pmatrix} \mathbf{2} & \mathbf{2} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

19. Which of the following can be calculated from the given matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$,
(i) A^2 (ii) B^2 (iii) AB (iv) BA
(A) (i) and (ii) only (B) (ii) and (iii) only **(C) (ii) and (iv) only** (D) all of these

20. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$. Which of the following statements are correct?
(i) $AB + C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ (ii) $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$ (iii) $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ (iv) $(AB)C = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$
(A) (i) and (ii) only (B) (ii) and (iii) only (C) (iii) and (iv) only (D) all of these

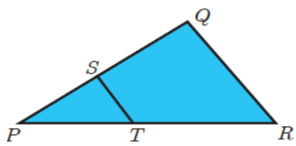
UNIT.IV GEOMETRY

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when
(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ **(C) $\angle B = \angle D$** (D) $\angle A = \angle F$

2. In ΔLMN, $\angle L = 60°$, $\angle M = 50°$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is
(A) 40° **(B) 70°** (C) 30° (D) 110°

3. If ΔABC is an isosceles triangle with $\angle C = 90°$ and $AC = 5\ cm$, then AB is
(A) 2.5 cm (B) 5 cm (C) 10 cm **(D) $5\sqrt{2}\ cm$**

4. In a given figure $ST \parallel QR$, $PS = 2\ cm$ and $SQ = 3\ cm$. Then the ratio of the area of ΔPQR to the area of ΔPST is
(A) 25 : 4 (B) 25 : 7 (C) 25 : 11 (D) 25 : 13

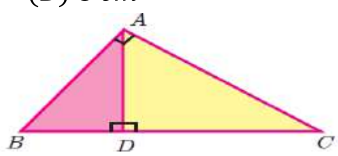


5. The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If $PQ = 10\ cm$, then the length of AB is
(A) $6\frac{2}{3}\ cm$ (B) $\frac{10\sqrt{6}}{3}\ cm$ (C) $66\frac{2}{3}\ cm$ **(D) 15 cm**

6. If in ΔABC, DE ∥ BC. $AB = 3.6\ cm$, $AC = 2.4\ cm$ and $AD = 2.1\ cm$ then the length of AE is
(A) 1.4 cm (B) 1.8 cm (C) 1.2 cm (D) 1.05 cm

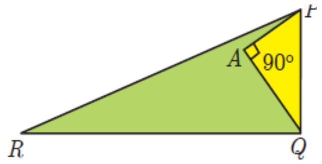
7. In a ΔABC, AD is the bisector of $\angle BAC$. If $AB = 8\ cm$, $BD = 6\ cm$ and $DC = 3\ cm$. The length of the side AC is (A) 6 cm **(B) 4 cm** (C) 3 cm (D) 8 cm

8. In the adjacent figure $\angle BAC = 90°$ and $AD \perp BC$ then
(A) $BD \cdot CD = BC^2$ (B) $AB \cdot AC = BC^2$
(C) $BD \cdot CD = AD^2$ (D) $AB \cdot AC = AD^2$



9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?
(A) 13 m (B) 14 m (C) 15 m (D) 12.8 m

10. In the given figure, $PR = 26$ cm, $QR = 24\ cm$, $\angle PAQ = 90°$, $PA = 6$ cm and $QA = 8\ cm$. Find $\angle PQR$
(A) 80° (B) 85° (C) 75° **(D) 90°**

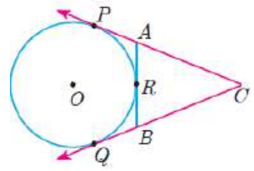


11. A tangent is perpendicular to the radius at the
(A) centre **(B) point of contact** (C) infinity (D) chord

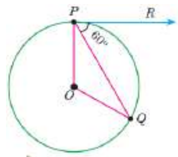
12. How many tangents can be drawn to the circle from an exterior point?
(A) one **(B) two** (C) infinite (D) zero

13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70°$ then the value of $\angle AOB$ is
(A) 100° **(B) 110°** (C) 120° (D) 130°

14. In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If $CP = 11\ cm$ and $BC = 7\ cm$, then the length of BR is
(A) 6 cm (B) 5 cm (C) 8 cm **(D) 4 cm**



15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is
(A) 120° (B) 100° (C) 110° (D) 90°



UNIT.V COORDINATE GEOMETRY

1. The area of triangle formed by the points $(-5, 0)$, $(0, -5)$ and $(5, 0)$ is
(A) 0 sq.units **(B) 25 sq.units** (C) 5 sq.units (D) none of these

2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is
(A) $x = 10$ (B) $y = 10$ (C) $x = 0$ (D) $y = 0$

3. The straight line given by the equation $x = 11$ is
(A) parallel to X axis **(B) parallel to Y axis**
(C) passing through the origin (D) passing through the point $(0, 11)$

4. If $(5, 7), (3, p)$ and $(6, 6)$ are collinear, then the value of p is
(A) 3 (B) 6 **(C) 9** (D) 12

5. The point of intersection of $3x - y = 4$ and $x + y = 8$ is
(A) (5, 3) (B) (2, 4) **(C) (3, 5)** (D) (4, 4)

6. The slope of the line joining $(12, 3)$, $(4, a)$ is $\frac{1}{8}$. The value of 'a' is
(A) 1 (B) 4 (C) -5 **(D) 2**

7. The slope of the line which is perpendicular to line joining the points $(0, 0)$ and $(-8, 8)$ is
(A) -1 **(B) 1** (C) $\frac{1}{3}$ (D) -8

8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then the slope of the perpendicular bisector of PQ is
(A) $\sqrt{3}$ **(B) $-\sqrt{3}$** (C) $\frac{1}{\sqrt{3}}$ (D) 0

9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is

(A) $8x + 5y = 40$ (B) $8x - 5y = 40$ (C) $x = 8$ (D) $y = 5$

10. The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is

(A) $7x - 3y + 4 = 0$ (B) $3x - 7y + 4 = 0$ **(C) $3x + 7y = 0$** (D) $7x - 3y = 0$

11. Consider four straight lines

(i) l_1: $3y = 4x + 5$ (ii) l_2: $4y = 3x - 1$ (iii) l_3: $4y + 3x = 7$ (iv) l_4: $4x + 3y = 2$

Which of the following statement is true ?

(A) l_1 and l_2 are perpendicular (B) l_1 and l_4 are parallel

(C) l_2 and l_4 are perpendicular (D) l_2 and l_3 are parallel

12. A straight line has equation $8y = 4x + 21$. Which of the following is true

(A) The slope is 0.5 and the y intercept is 2.6 (B) The slope is 5 and the y intercept is 1.6

(C) The slope is 0.5 and the y intercept is 1.6 (D) The slope is 5 and the y intercept is 2.6

13. When proving that a quadrilateral is a trapezium, it is necessary to show

(A) Two sides are parallel **(B) Two parallel and two non-parallel sides.**

(C) Opposite sides are parallel (D) All sides are of equal length

14. When proving that a quadrilateral is a parallelogram by using slopes you must find

(A) The slopes of two sides **(B) The slopes of two pair of opposite sides**

(C) The lengths of all sides (D) Both the lengths and slopes of two sides

15. $(2, 1)$ is the point of intersection of two lines

(A) $x - y - 3 = 0$; $3x - y - 7 = 0$ **(B) $x + y = 3$; $3x + y = 7$**

(C) $3x + y = 3$; $x + y = 7$ (D) $x + 3y - 3 = 0$; $x - y - 7 = 0$

UNIT.VI TRIGONOMETRY

1. The value of $sin^2\theta + \frac{1}{1+tan^2\theta}$ is equal to

(A) $tan^2\theta$ **(B) 1** (C) $cot^2\theta$ (D) 0

2. $tan\theta\, cosec^2\theta - tan\theta$ is equal to

(A) $sec\theta$ (B) $cot^2\theta$ (C) $sin\theta$ **(D) $cot\theta$**

3. If $(sin\alpha + cosec\alpha)^2 + (cos\alpha + sec\alpha)^2 = k + tan^2\alpha + cot^2\alpha$, then the value of k is equal to

(A) 9 **(B) 7** (C) 5 (D) 3

4. If $sin\theta + cos\theta = a$ and $sec\theta + cosec\theta = b$, then the value of $b(a^2 - 1)$ is equal to

(A) $2a$ (B) $3a$ (C) 0 (D) $2ab$

5. If $5x = sec\theta$ and $\frac{5}{x} = tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to

(A) 25 **(B) $\frac{1}{25}$** (C) 5 (D) 1

6. If $sin\theta = cos\theta$, then $2tan^2\theta + sin^2\theta - 1$ is equal to

(A) $\frac{-3}{2}$ **(B) $\frac{3}{2}$** (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

7. If $x = atan\theta$ and $y = bsec\theta$ then

(A) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1 + tan\,\theta + sec\,\theta)(1 + cot\theta - cosec\theta)$ is equal to

(A) 0 (B) 1 **(C) 2** (D) -1

9. $a\,cot\theta + b\,cosec\theta = p$ and $b\,cot\theta + a\,cosec\,\theta = q$ then $p^2 - q^2$ is equal to

(A) $a^2 - b^2$ **(B) $b^2 - a^2$** (C) $a^2 + b^2$ (D) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure
(A) 45° (B) 30° (C) 90° **(D) 60°**

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to (A) $\sqrt{3}\,b$ **(B) $\frac{b}{3}$** (C) $\frac{b}{2}$ (D) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m high. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to
(A) 41.92 m **(B) 43.92 m** (C) 43 m (D) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is
(A) 20, $10\sqrt{3}$ (B) 30, $5\sqrt{3}$ (C) 20, 10 **(D) 30, $10\sqrt{3}$**

14. Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is
(A) $\sqrt{2}\,x$ **(B) $\frac{x}{2\sqrt{2}}$** (C) $\frac{x}{\sqrt{2}}$ (D) $2x$

15. The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is
(A) $\frac{h(1+tan\beta)}{1-tan\beta}$ (B) $\frac{h(1-tan\beta)}{1+tan\beta}$ (C) $h\,tan(45°-\beta)$ (D) none of these

UNIT.VII MENSURATION

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is
(A) $60\pi\ cm^2$ (B) $68\pi\ cm^2$ (C) $120\pi\ cm^2$ **(D) $136\pi\ cm^2$**

2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
(A) $4\pi r^2$ sq. units (B) $6\pi r^2$ sq. units (C) $3\pi r^2$ sq. units (D) $8\pi r^2$ sq. units

3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be
(A) 12 cm (B) 10 cm (C) 13 cm (D) 5 cm

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
(A) 1 : 2 **(B) 1 : 4** (C) 1 : 6 (D) 1 : 8

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is
(A) $\frac{9\pi h^2}{8}$ sq. units (B) $24\pi h^2$ sq. units **(C) $\frac{8\pi h^2}{9}$ sq. units** (D) $\frac{56\pi h^2}{9}$ sq. units

6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is
(A) $5600\pi\ cm^3$ **(B) $1120\pi\ cm^3$** (C) $56\pi\ cm^3$ (D) $3600\pi\ cm^3$

7. If the radius of the base of a cone is tripled and the height is doubled then the volume is
(A) made 6 times **(B) made 18 times** (C) made 12 times (D) unchanged

8. The total surface area of a hemi-sphere is how much times the square of its radius.
(A) π (B) 4π **(C) 3π** (D) 2π

9. A solid sphere of radius $x\ cm$ is melted and cast into a shape of a solid cone of same radius. The height of the cone is
(A) $3x\ cm$ (B) $x\ cm$ **(C) $4x\ cm$** (D) $2x\ cm$

10. A frustum of a right circular cone is of height $16cm$ with radii of its ends as $8cm$ and $20cm$. Then, the volume of the frustum is
(A) $3328\,\pi\ cm^3$ (B) $3228\,\pi\ cm^3$ (C) $3240\,\pi\ cm^3$ (D) $3340\,\pi\ cm^3$

11. A shuttle cock used for playing badminton has the shape of the combination of
(A) a cylinder and a sphere (B) a hemisphere and a cone
(C) a sphere and a cone **(D) frustum of a cone and a hemisphere**

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is **(A) 2 : 1** (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius $1\ cm$ and height $5\ cm$ is **(A) $\frac{4}{3}\pi$** (B) $\frac{10}{3}\pi$ (C) $5\,\pi$ (D) $\frac{20}{3}\pi$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is (A) 1 : 3 **(B) 1 : 2** (C) 2 : 1 (D) 3 : 1

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is (A) 1 : 2 : 3 (B) 2 : 1 : 3 (C) 1 : 3 : 2 **(D) 3 : 1 : 2**

UNIT.VIII STATISTICS AND PROBABILITY

1. Which of the following is not a measure of dispersion?
(A) Range (B) Standard deviation **(C) Arithmetic mean** (D) Variance

2. The range of the data 8, 8, 8, 8, 8...8 is
(A) 0 (B) 1 (C) 8 (D) 3

3. The sum of all deviations of the data from its mean is
(A) Always positive (B) always negative **(C) zero** (D) non-zero integer

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all observations is (A) 40000 **(B) 160900** (C) 160000 (D) 30000

5. Variance of first 20 natural numbers is (A) 32.25 (B) 44.25 **(C) 33.25** (D) 30

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is
(A) 3 (B) 15 (C) 5 **(D) 225**

7. If the standard deviation of x, y, z is p then the standard deviation of $3x + 5$, $3y + 5$, $3z + 5$ is
(A) $3p + 5$ **(B) $3p$** (C) $p + 5$ (D) $9p + 15$

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is
(A) 3.5 (B) 3 (C) 4.5 (D) 2.5

9. Which of the following is incorrect?
(A) $P(A) > 1$ (B) $0 \le P(A) \le 1$ (C) $P(\emptyset) = 0$ (D) $P(A) + P(\bar{A}) = 1$

10. The probability of a red marble selected at random from a jar containing p red, q blue and r green marbles is (A) $\frac{q}{p+q+r}$ **(B) $\frac{p}{p+q+r}$** (C) $\frac{p+q}{p+q+r}$ (D) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is (A) $\frac{3}{10}$ **(B) $\frac{7}{10}$** (C) $\frac{3}{9}$ (D) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is (A) 2 **(B) 1** (C) 3 (D) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is

(A) 5 (B) 10 **(C) 15** (D) 20

14. If a letter is chosen at random from the English alphabets $\{a, b, \ldots, z\}$, then the probability that the letter chosen precedes x (A) $\frac{12}{13}$ (B) $\frac{1}{13}$ **(C) $\frac{23}{26}$** (D) $\frac{3}{26}$

15. A purse contains 10 notes of Rs.2000, 15 notes of Rs.500, and 25 notes of Rs.200. One note is drawn at random. What is the probability that the note is either a Rs.500 note or Rs.200 note?

(A) $\frac{1}{5}$ (B) $\frac{3}{10}$ (C) $\frac{2}{3}$ **(D) $\frac{4}{5}$**

GEOMETRY & GRAPH

QUESTION BANK-2022

GEOMETRY – Constructions

I. SIMILAR TRIANGLES :- (Big to Small)

1. Construct a triangle similar to a given triangle with its sides equal to – of the corresponding sides of the triangle (scale factor – 1)

2. Construct a triangle similar to a given triangle with its sides equal to – of the corresponding sides of the triangle (scale factor –)

3. Construct a triangle similar to a given triangle with its sides equal to – of the corresponding sides of the triangle (scale factor –)

II. SIMILAR TRIANGLES :- (Small to Big)

4. Construct a triangle similar to a given triangle with its sides equal to – of the corresponding sides of the triangle (scale factor – 1)

5. Construct a triangle similar to a given triangle with its sides equal to – of the corresponding sides of the triangle (scale factor –)

6. Construct a triangle similar to a given triangle with its sides equal to – of the corresponding sides of the triangle (scale factor –)

III. TRIANGLES :- (When **MEDIAN** is given)

7. Construct a ΔPQR in which and the **median** from to is 5.8 cm. Find the length of the ***altitude*** from to PQ

8. Construct a ΔPQR in which and the **median** from to is 4.4 cm. Find the length of the ***altitude*** from to QR

9. Construct a in which the base and the **median** from to is 6 cm.

IV. TRIANGLES :- (When ALTITUDE is given)

10. Construct a triangle such that and the **altitude** from to is of length 4.2 cm.

11. Construct a ΔPQR such that and the **altitude** from to is of length 4.5 cm.

12. Construct a triangle such that and the **altitude** from to AB is 4 cm.

V. TRIANGLES :- (When the point of ANGLE BISECTOR is given)

13. Draw a triangle of base and the **bisector** of meets at D such that 6 cm.

14. Draw a triangle of base and the **bisector** of meets at D such that 4 cm.

15. Draw ΔPQR such that vertical angle and the **bisector** of the vertical angle meets the base at where 5.2 cm.

VI. TANGENTS TO A CIRCLE: (Using the Centre)

16. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

17. Draw a tangent at any point on the circle of radius 3.4 cm and centre at ?

VII. TANGENTS TO A CIRCLE: (Using Alternate Segment Theorem)

18. Draw a circle of radius 4 cm. At a point on it draw a tangent to the circle using the **alternate−segment theorem**.

19. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the **alternate – segment theorem**.

VIII. TANGENTS TO A CIRCLE: (Pair of Tangents or Two Tangents)

20. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. **Draw the two tangents** and to the circle and measure their lengths.

21. **Draw the two tangents** from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

22. **Draw the two tangents** from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.

23. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and **draw the two tangents** to the circle from the point.

24. **Draw a tangent** to the circle from the point having radius 3.6 cm, and centre at point is at a distance 7.2 cm from the centre.

GRAPH

I. GRAPH of VARIATION :- (Direct Variation)

1. Varshika drew 6 circles with different sizes. Draw a graph for the relationship betweem the diameter and circumference of each circle (approximately) as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x) **cm**	1	2	3	4	5
Circumference (y) **cm**	3.1	6.2	9.3	12.4	15.5

2. A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find (i) the constant of variation
   (ii) how far will it travel in 90 minutes
   (iii) the time required to cover a distance of 300 km from the graph.

3. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find (i) the marked price when a customer gets a discount of Rs.3250 (from Graph)
   (ii) the discount when the marked price is Rs.2500

4. Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also, (i) find y when $x = 9$ (ii) find x when $y = 7.5$

5. A two wheeler parking zone near bus stand charges as below:

Time (in hours) (x)	4	8	12	24
Amount Rs. (y)	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also, (i) find the amount to be paid when parking time is 6 hrs; (ii) find the parking duration when the amount paid is Rs.150.

II. GRAPH of VARIATION :- (Inverse Variation)

6. A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below:

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

(i) Graph the above data and identify the type of variation.
(ii) From the graph, find the number of days required to complete the work if the company decided to opt for 120 workers?
(iii) If the work has to be completed by 200 days, how many workers are required?

7. Nishanth is the winner in a Marathan race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/kr. And, they have covered the distance in 2 hrs, 3 hrs, 4 hrs and 6 hrs respectively. Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

8. Draw the graph of $xy = 24$, $x, y > 0$. Using the graph find, (i) y when $x = 3$ and (ii) find x when $y = 6$.

9. The following table shows the data about the number of pipes and the time taken to fill the same tank

No. of pipes (x)	2	3	6	9
Time taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence
(i) Find the time taken to fill the tank when five pipes are used
(ii) Find the number of pipes when the time is 9 minutes

10. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as shown below

No. of participants (x)	2	4	6	8	10
Amount for each participant in Rs. (y)	180	90	60	45	36

(i) Find the constant of variation.
(ii) Graph the above data. Hence, find how much will each participant get if the number of participants are 12.

III. NATURE of the SOLUTIONS :- (Graphically)

Discuss the **nature of solutions** of the following **quadratic equations**

11. $x^2 + x - 12 = 0$
12. $x^2 - 8x + 16 = 0$
13. $x^2 + 2x + 5 = 0$

Graph the following **quadratic equations** and state its **nature of solutions:**

14. $x^2 - 9x + 20 = 0$
15. $x^2 - 4x + 4 = 0$
16. $x^2 + x + 7 = 0$
17. $x^2 - 9 = 0$
18. $x^2 - 6x + 9 = 0$
19. $(2x - 3)(x + 2) = 0$

IV. Solving QUADRATIC EQUATIONS :- (Through intersection of lines)

20. Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$.
21. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$.
22. Draw the graph of $y = x^2 + 4x + 3$ and hence find the roots of $x^2 + x + 1 = 0$.
23. Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.
24. Draw the graph of $y = x^2 - 4x + 3$ and use it to solve $x^2 - 6x + 9 = 0$.
25. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.
26. Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$.
27. Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$.
28. Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$.
29. Draw the graph of $y = 2x^2 - 3x - 5$ and hence use it to solve $2x^2 - 4x - 6 = 0$
30. Draw the graph of $y = (x - 1)(x + 3)$ and hence use it to solve $x^2 - x - 6 = 0$

Relations and Functions

(2 Mark questions)

1.$A = \{2,-2,3\}, B = \{1,-4\}$ then find $A\times B, A\times A$. (Exercise 1.1- 1 (i))

Solution:

$A\times B = \{2,-2,3\} \times \{1,-4\}$

$= \{(2,1), (2,-4), (-2,1), (-2,-4), (3,1), (3,-4)\}$

$A\times A = \{2,-2,3\}\times\{2,-2,3\}\{(2,2), (2,-2), (2,3), (-2,2), (-2,-2),$

$(-2,3), (3,2), (3,-2), (3,3)\}$

2. If $A = B = \{p.q\}$ then find $A \times B, B \times A$ (Exercise 1.1-1(ii))

Solution:

$A\times B = \{p,q\}\times\{p,q\}$

$= \{(p,p), (p,q), (q,p), (q,q)\}$

$B\times A = \{p,q\}\times\{p,q\}$

$= \{(p,p), (p,q), (q,p), (q,q)\}$

3.If $A = \{m,n\}, B = \phi$ then find $A\times B, A\times A, B\times A$. (Exerxcise 1.1-1(iii))

Solution:

$A\times B = \{m,n\}\times\{\} = \{\ \}$

$A\times A = \{m,n\}\times\{m,n\}$

$= \{(m,m), (m,n), (n,m), (n,n)\}$

$B\times A = \{\ \ \}\times\{m,n\} = \{\ \ \}$

4.A={1,2,3}, B={x | x is a prime number less than 10} then $A\times B$, $B\times A$. (Exercise 1.1-2)

Solution:

$A = \{1,2,3\}\ B = \{2,3,5,7\}$

$A\times B = \{1,2,3\}\times(2,3,5,7\}$

$= (1,2),(1,3),(1,5),(1,7),(2,2),(2,3),$

$(2,5)(2,7),(3,2),(3,3),(3,5),(3,7)\}$

$B\times A = \{2,3,5,7\}\times\{1,2,3\}$

$= \{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(5,1),$

$(5,2),(5,3),(7,1),(7,2),(7,3)\}$

Solution:

4.A={1,2,3}, B={x | x is a prime number less than 10} then $A\times B$, $B\times A$. (Exercise 1.1-2)

Solution:

$A = \{1,2,3\}\ B = \{2,3,5,7\}$

$A\times B = \{1,2,3\}\times(2,3,5,7\}$

$= (1,2),(1,3),(1,5),(1,7),(2,2),(2,3),$

$(2,5)(2,7),(3,2),(3,3),(3,5),(3,7)\}$

$B\times A = \{2,3,5,7\}\times\{1,2,3\}$

$= \{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(5,1),$

$(5,2),(5,3),(7,1),(7,2),(7,3)\}$

5.If $A = \{1,3,5\}, B=\{2,3\}$ then i) Find $A\times B, B\times A$. ii) $A\times B = B\times A$ if not why? iii) Show that. $n(A\times B) = n(B\times A) = n(A) \times n(B)$. (Example-1.1)

Solution:

$A\times B = \{1,3,5\}\times\{2,3\}$

$=\{(1,2), (1,3), (3,2), (3,3),(5,2),(5,3)\}$ .... (1)

$B\times A = \{2,3\}\times\{1,3,5\}$

$=\{(2,1),(2,3),(2,5),(3,1),(3,3),(3,5)\}$ ....(2)

from (1) and (2) $A\times B \neq B\times A$ because $(1,2) \neq (2,1)$ ....

$n(A\times B)=6, n(B\times A) = 6$

$n(A)=2,\ n(B) = 3$

$\therefore n(A\times B)=n(B\times A) = n(A)\times n(B)$

$6 = 6 = 2\times 3$

$6 = 6 = 6$

6.$A\times B = \{(3,2), (3,4), (5,2), (5,4)\}$ then A and B. (example.1.2)

Solution:

A = {Set of all first coordinates of to elements $A\times B$}

A = {3,5}

B = {Set of all second coordinates of elements of $A\times B$}

B={2,4}

7.$B\times A = \{(-2,-3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$ find A and B.(exercise 1.1 – 3)

A = {Set of all second coordinates of elements $B \times A$}

A = {3,4}

B = {Set of all first coordinates of elements of $B \times A$}

B = {-2,0,3}

8.For Practice

If A={5,6}, B={4,5,6}, C={5,6,7} show that $A \times A = (B \times B) \cap (C \times C)$. (Exercise 1.1-4).

9. Let A = {3,4,7,8}, B={1,7,10} which of the followings sets are relations from A to B? (எ.கா.1.4)

(i) R_1={(3,7), (4,7), (7,10),(8,1)}

(ii) R_2 = {(3,1), (4,12)}

Solution:

$$A \times B = \{3,4,7,8\} \times \{1,7,10\}$$
$$= \{(3,1),(3,7),(3,10),(4,1),(4,7),(4,10),$$
$$(7,1),(7,7),(7,10),(8,1),(8,7),(8,10)\}$$

i) $R_1 \subset A \times B$

R_1 is relation from A to B.

ii) $(4,12) \in R_2$ but $(4,12) \notin A \times B$

R_2 is not relation from A to B.

10.Let A = {1,2,3,7}, B = {3,0,-1,7} then $R_1 = \{(2,-1),(7,7),(1,3)\}$ is a relation from A to B? (Exercise 1.2-1)

Solution:

$$A \times B = \{1,2,3,7\} \times \{3,0,-1,7\}$$
$$= \{(1,3),(1,0),(1,-1),(1,7),(2,3),(2,0),$$
$$(2,-1),(2,7),(3,3),(3,0),(3,-1),$$
$$(3,7),(7,3),(7,0),(7,-1),(7,7)\}$$
$$R_1 = \{(2,-1),(7,7)(1,3)\}$$
$$R_1 \subset A \times B$$

R_1 is a relation from A to B.

11.A relation R is given by the set {(x,y) / y=x+3, $x \in \{0,1,2,3,4,5\}$ Determine its domain and range (exercise 1.2 -3)

Solution:

$$x = 0 \Rightarrow y = 0+3 = 3;$$
$$x = 1, y = 1+3 = 4$$
$$x = 2 \Rightarrow y = 2+3 = 5;$$
$$x = 3, y = 3+3 = 6$$
$$x = 4 \Rightarrow y = 4+3 = 7;$$
$$x = 5, y = 5+3 = 8$$
$$R = \{(0,3),(1,4),(2,5),(3,6),(4,7),(5,8)\}$$

Domain = {0,1,2,3,4,5}

Range = {3,4,5,6,7,8}

12.Let A= {1,2,3,4,...,45} and R be the relation defined as is square of a number A. Write R as a subset of A ×A. Also find the domain and range of R.

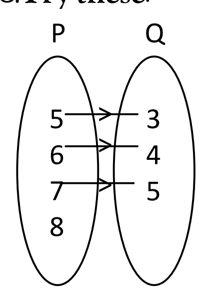
Solution:

R = { 1,4,9,16,25,36} $R \subset A \times B$

Domain = {1,2,3,4,5,6}

Range = {1,4,9,16,25,36}.

13.Try these:



The arrow diagram shows a relationship between the sets P and Q. Write the relation is (i) set builder from 9ii) Roster form (iii) what is the domain and range of R. (Example 1.5)

Solution:

i) Set builder form of R

$\{(x, y / y = x-2, x \in p, y \in q\}$

ii) Roster from of R= {(5,3), (6,4), (7,5)}

iii) Domain = {5,6,7}, Range = {3,4,5}

14.Let X={3,4,6,8}, $R = \{(x, f(x)) \mid x \in x, f(x) = x^2 + 1\}$ is a function from X to N? (Exercise 1.3-2)

Solution:

$$x = 3 \Rightarrow f(3) = 3^2 + 1 = 9 + 1 = 10$$
$$x = 4 \Rightarrow f(4) = 4^2 + 1 = 16 + 1 = 17$$
$$x = 6 \Rightarrow f(6) = 6^2 + 1 = 36 + 1 = 37$$
$$x = 8 \Rightarrow f(8) = 8^2 + 1 = 64 + 1 = 65$$

R={(3,10),(4,17),(6,37), (8,65)}

All elements if x have only one images in y. ∴ R is a function.

15.X = {1,2,3,4}, Y = { 2,4,6,810} and R= {(1,2),(2,4),(3,6),(4,8)} show that R is a function and find its domain, Co-domain and range. (Example 1.6)

Solution:

All elements in x have only on images in Y ∴ R is a function.

Domain X={ 1,2,3,4}

Co-domain Y={2,4,6,8,10}

Range ={2,4,6,8}

16.A relation f is defined $f(x) = x^2 - 2$ where $x \in \{-2, -1, 0, 3)$ (i) list of elements of f. ii) Is f a function? (example .1.7)

Solution:

(i) $f(x) = x^2 - 2, x \in \{-2, -1, 0, 3\}$

$$x = -2 \Rightarrow f(-2) = (-2)^2 - 2 = 4 - 2 = 2$$
$$x = -1 \Rightarrow f(-1) = (-1)^2 - 2 = 1 - 2 = -1$$
$$x = 0 \Rightarrow f(0) = (0)^2 - 2 = 0 - 2 = -2$$
$$x = 3 \Rightarrow f(3) = (3)^3 - 2 = 9 - 2 = 7$$

∴ f ={(-2,2), (-1,-1), (0,2), (3,7)}

(ii) Each elements is the domain of f has a unique images.

∴ f is function

17.Let $f(x) = 2x + 5$ if $x \neq 0$ then find $\frac{f(x+2) - f(2)}{x}$

(Exercise 1.3-5)

Solution:

$$f(x) = 2x + 5$$
$$f(x+2) = 2(x+2) + 5$$
$$= 2x + 4 + 5$$
$$f(x+2) = 2x + 9$$
$$f(x) = 2x + 5$$
$$f(2) = 2(2) - 5$$
$$= 4 + 5$$
$$f(2) = 9$$
$$\frac{f(x+2) - f(2)}{x} = \frac{2x + 9 - 9}{x} = \frac{2x}{x} = 2$$

18Given $f(x) = 2x - x^2$ find $f(x) + f(1)$.

Solution:

$$f(x) + f(1) = 2x = x^2 + 2(1) - (1)^2$$
$$= 2x - x^2 + 2 - 1$$
$$= 2x - x^2 + 1$$

19.A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = \{f(x))^2$ (Exercise 1.3-8).

Solution:

$$f(x2) = \{f(x))^2$$
$$3 - 2x^2 = (3 - 2x)^2$$
$$3 - 2x^2 = 9 + 4x^2 - 12x$$
$$3 - 2x^2 - 9 - 4x^2 + 12x = 0$$
$$-6x^2 + 12x - 6 = 0 \Rightarrow 6x^2 - 12x + 6 = 0$$
$$\div 6, x^2 - 2x + 1 = 0 \quad (x-1)(x-1) = 0$$
$$x = 1, 1$$

20.A plane is flying at speed of 500 km per hour. Express the distance 'd' travelled by the plane as function to time t in hours.

Solution:

Distance = time x Speed

d=500t

21.For practice, $f = \{(x, y)/x, y \in N^2 \text{and } y = 2x\}$ be a relation on N. Find the domain, co-domain, and range. Is this relation a function? (Exercise 1.3-1)

22.Show that the function $f: N \rightarrow N$ defind $f(x) = 2x - 1$ is one – one but not on to. (Exercise 1.4-4)

Solution:

$f: N \rightarrow N \quad f(x) = 2x - 1$

$x = 1 \Rightarrow f(1) = 2(1) - 1 = 2 - 1 = 1$

$x = 2 \Rightarrow f(2) = 2(2) - 1 = 4 - 1 = 3$

$x = 3 \Rightarrow f(3) = 2(3) - 1 = 6 - 1 = 5$

$x = 4 \Rightarrow f(4) = 2(4) - 1 = 8 - 1 = 7$..........

Every elements in N have only one image in N

∴ f is one – one function

Range ≠ Co-domain in N

∴ f is not one to function

23.Show that the function $f: N \rightarrow N$ defined by $f(m) = m^2 + m + 3$ is one – on function.

Solution:

$f: N \rightarrow N \quad f(m) = m^2 + m + 3$

$m = 1 \Rightarrow f(1) = 1^2 + 1 + 3 = 1 + 1 + 3 = 5$

$m = 2 \Rightarrow f(2) = 2^2 + 2 + 3 = 4 + 2 + 3 = 9$

$m = 3 \Rightarrow f(3) = 3^2 + 3 + 3 = 9 + 3 = 15$

$m = 4 \Rightarrow f(4) = 4^2 + 4 + 3 = 16 + 4 + 3 = 23$........

Every elements in N have only one image in N.

∴ f is one – one function.

24.Let A = { 1,2,3,4) and B=N. Let $f: A \rightarrow B$ be defined by $f(x)=x^3$ then, (i) find the range of f. (ii) identify the type of function.

Solution:

A = {1,2,3,4} $f(x)=x^3$

$x = 1 \Rightarrow f(1) = 1^3 = 1$

$x = 2 \Rightarrow f(2) = 2^3 = 8$

$x = 3 \Rightarrow f(3) = 3^3 = 27$

$x = 4 \Rightarrow f(4) = 4^3 = 64$

Range = {1,8,27,64}

Each elements in a have only one image on B

∴ f is one – one function.

25.Let f be a function $f: N \rightarrow N$ be defined by f(x)=3x+2,

(i) find the image of 1,2,3.

(ii) find the pre-images 29, 53

(iii) Identify the type of function. (example 1.15)

$f: N \rightarrow N \quad f(x) = 3x + 2$

$x = 1 \Rightarrow f(1) = 3(1) + 2 = 3 + 2 = 5$

$x = 2 \Rightarrow f(2) = 3(2) + 2 = 6 + 2 = 8$

$x = 3 \Rightarrow f(3) = 3(3) + 2 = 9 + 2 = 11$

(i) The images of 1,2,3 are 5,8,11 respectively.

(ii)f(x) = 29

$3x + 2 = 29 \Rightarrow x = 9$

Pre-image of 29 is 9

$f(x) = 53$

$3x + 2 = 53 \Rightarrow x = 17$

Pre-image of 53 = 17

Pre image of 53 is 17.

(iii) Since difference elements of N have different images in the co-domain, the function f is one – one function range f = { 5,8,11,14,17...} is a proper subset of N / ∴ f is an into function

Thus f is one – one and into function.

26. Let f be a function from R to R defind by, $f(x) = 3x - 5$ Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f. (example 1.17)

Solution:

$(a, 4)$ then f(a)=4

$$f(a) = 4$$
$$3a - 5 = 4$$
$$\Rightarrow 3a = 9 \Rightarrow a = 3$$

3a-5=4

$(1, b)$ எனில் $f(1) = b$

$$3(1) - 5 = b \Rightarrow b = -2$$

27. $f(x) = 3x + 2, g(x) = 6x - k$ and if $f \ o \ g = g \ o \ f$ then find the value of K. (Example 1.21)

Solution:

$$f \ o \ g\ (x) = g \ o \ f\ (x)$$
$$f[g(x)] = g[f(x)]$$
$$f(2x + k) = g(3x - 2)$$
$$3(2x + k) - 2 = 2(3x - 2) + k$$
$$6x + 3k - 2 = 6x - 4 + k$$
$$3k - k = -4 + 2$$
$$2k = -2 \Rightarrow k = -1$$

28. $f(x) = 3x + 2, g(x) = 6x - k$ and if $f \ o \ g = g \ o \ f$ then find the value of K. (Exercise 1.5-2)

$$fog(x) = gof(x)$$
$$f[g(x)] = g[f(x)]$$
$$f(6x - k) = g(3x + 2) - k$$
$$18x - 3k + 2 = 18x + 12 - k$$
$$-3k + k = 12 - 2$$
$$-2k = 10 \Rightarrow k = -5$$

Solution:

29. Find k if $f \ o \ f(k) = 5$ where $f(k) = 2k - 1$ then find the value of k.(Example 1.22)

Solution:

$$f \ o \ f(k) = 5$$
$$f[2k - 1] = 5$$
$$2(2k - 1) - 1 = 5$$
$$4k - 2 - 1 = 5$$
$$4k = 8$$
$$k = 2$$

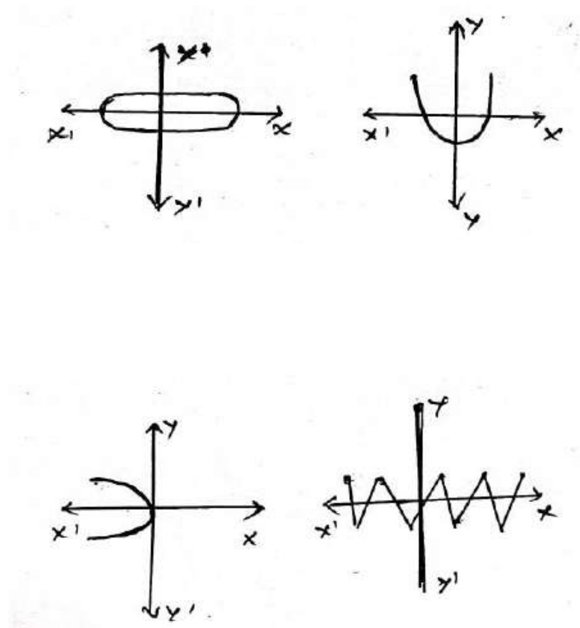
30. Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions. (Example 1.20)

Solution:

$f_2(x) = 2x^2 - 5x + 3$ and $f_1(x) = \sqrt{x}$

$$f(x) = \sqrt{2x^2 - 5x + 3}$$
$$= \sqrt{f_2(x)}$$
$$= f_1[f_2(x)]$$
$$= f_1 \ o \ f_2(x)$$
$$= f_1 \ o \ f_2$$

31. Determine which of the following curves represent of function? (Example 1.10)



The curves in fig (i), (iii) don not represent of function as the vertical lines meet the curves in two points.

The curves in fig (ii), (iv) represent a function as the vertical lines meet the curve in a at most one point.

5 Marks

1. Let $A=\{x \in N / 1 < x < 4\}, B=\{x \in W / 0 \le x < 2\}$, $C=\{x \in N / x < 3\}$ Then vertify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (Example 1.3)

Solution:

$A=\{x \in N / 1 < x < 4\}$; $A=(2,3)$

$B=\{x \in W / 0 \le x < 2\}$; $B=(0,1)$

$C=\{x \in N / x < 3\}$ $C=(1,2)$

(i) LHS

$(B \cup C) = \{0,1\} \cup \{1,2\}$

$= \{0,1,2\}$

$A \times (B \cup C) = \{(2,0),(2,1),(2,2),(3,0),$
$(3,1),(3,2)\}.....(1)$

RHS

$(A \times B) = \{2,3\} \times \{0,1\}$

$= \{(2,0),(2,1),(3,0),(3,1)\}$

$(A \times C) = \{2,3\} \times \{2,1\}$

$= \{(2,1),(2,2)(3,1),(3,2)\}$

$(A \times B) \cup (A \times C) = \{(2,0),(2,1),(3,0),(3,1)\}$
$\cup \{(2,1),(2,2),(3,1),(3,2)\}$
$= \{(2,.0),(2,1),(2,2),(3,0),$
$(3,1),(3,2)\}...(2)$

$(1) = (2)$

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$

RHS

(ii) LHS

$(B \cap C) = \{0,1\} \cap \{1,2\}$

$= \{1\}$

$A \times (B \cap C) = \{2,3\} \times \{1\}$

$= \{(2,1),(3,1)\}....(1)$

$(A \times B) = \{2,3\} \times \{0,1\}$

$= \{(2,0),(2,1)(3,0)(3,1)\}$

$(A \times C) = \{2,3\} \times \{1,2\}$

$= \{(2,1),(2,2),(3,1),(3,2)\}$

$(A \times B) \cap (A \times C) = \{(2,1),(3,1)\}.....(2)$

$(1) = (2)$

$\therefore A \times (B \cap C) = (A \times B) \cap (A \times c)$

3. Let $A=\{x \in W / x < 2\}, B=\{x \in N / 1 < x \le 4\}$, $C=\{3,5\}$ Verify that (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$ (For practice) (Exercise 1.1 -6)

Solution:

$A = \{x \in W / x < 2\} = \{0,1\}$

$B = \{x \in N / 1 < x \le 4\} = \{2,3,4\}$

$C = \{3,5\}$

(i)LHS

$(B \cup C) = \{2,3,4\} \cup \{3,5\}$

$= \{2,3,4,5\}$

$A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$

$= \{(0,2),(0,3),(0,4),(0,5),(1,2),$
$(1,3)(1,4),(1,5)\}..............(1)$

$A\times B=\{(0,1)\}\times\{2,3,4\}$

$=\{(2,0),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A\times C=\{0,1\}\times\{3,5\}$

$=\{(0,3),(0,5),(1,3),(1,5)\}$

$(A\times B)\cup(A\times C)=\{(0,2),(0,3),(0,4),(0,5),(1,2),$
$(1,3),(1,4),(1,5)\}.......(2)$

(1) = (2)

$$A\times(B\cup C)=(A\times B)\cup(A\times C)$$

(ii)LHS

$(B\cap C)=\{2,3,4\}\cap\{3,5\}$

$=\{3\}$

$A\times(B\cap C)=\{0,1\}\times\{3\}$

$=\{(0,3),(1,3)\}..\_\_\_\_.(1)$

RHS

$(A\times B)=\{0,1\}\times\{2,3,4\}$

$=\{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$(A\times C)=\{0,1\}\times[3,5]$

$=\{(0,3),(0,5),(1,3),(1,5)\}.$

$(A\times B)\cap(A\times C)=\{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}\cap$
$\{(0,3),(0,5),(1,3),(1,5)\}$
$=\{(0,3),(1,3)\}.............(2)$

(1) = (2)

$$A\times(B\cap C)=(A\times B)\cap(A\times C)$$

3.Let A is set of all natural numbers less than 8, B is set of all prime numbers less that 8, C is set of even prime number, Verify that

(i) $(A\cap B)\times C=(A\times C)\cap(B\times C)$

(ii) $A\times(B-C)=(A\times B)-(A\times C)$

(exercise 1.7)

Solution:

$A=\{1,2,3,4,5,6,7\}$

$B=\{2,3,5,7\}$

$C=\{2\}$

(i)LHS

$(A\cap B)=\{1,2,3,4,5,6,7\}\cap\{2,3,5,7\}$

$=\{2,3,5,7\}$

$(A\cap B)\times C=\{2,3,5,7\}\times\{2\}$

$=\{(2,2),(3,2),(5,2),(7,2)\}.....(1)$

RHS

$(A\times C)=\{1,2,3,4,5,6,7\}\times\{2\}$

$=\{(1,2),(2,.2),(3,2),(4,2),(5,2),(6,2),$
$(7,2)\}$

$(B\times C)=\{2,3,5,7\}\times\{2\}$

$=\{(2,2),(3,2),(5,2)(7,2)\}$

$(A\times C)\cap(B\times C)=\{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}\cap$
$\{(2,2),(3,2),(5,2),(7,2)\}$
$=\{(2,2),(3,2),(5,2),(7,2)\}....(2)$

(1) = (2)

$$(A\cap B)\times C=(A\times C)\cap(B\times C)$$

(ii)LHS

$(B-C)=\{2,3,5,7\}-\{2\}$

$=\{3,5,7\}$

$A\times(B-C)=\{1,2,3,4,5,6,7\}\times\{3,5,7\}$

$=\{(1,3),(1,5),(1,7),(2,3),(2,5),(2,7),(3,3),(3,5),$
$(3,7),(4,3),(4,5),(4,7),(5,3),(5,5),(5,7),(6,3),$
$(6,5),(6,7),(7,3),(7,5),(7,7)\}......(1)$

RHS

$(A\times B)=\{1,2,3,4,5,6,7\}\times(2,3,5,7\}$

$=\{(1,2),(1,3),(1,5),(1,7),(2,2),(2,2),(2,3),(2,5),(2,7),$
$(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),(4,7),(5,2),$
$(5,3),(5,5),(5,7),,(6,2),(6,3),(6,5),(6,7),(7,2),(7,3),$
$(7,5),(7,7)\}$

$(A\times C)=\{1,2,3,4,5,6,7\}\times\{2\}$

$=\{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,.2)\}$

$(A\times B)-(A\times C)$

$=\{(1,3),(1,5),(1,7),(2,3),(2,5),2,7),(3,3),(3,5),(3,7)$
$(4,3),(4,5),(4,7),(5,3),(5,5),(5,7),(6,3),(6,5),(6,7)$
$(7,3),(7,5),(7,7)\}......(2)$

(1) =(2)

$$A\times(B-C)=(A\times B)-(A\times C)$$

4. $\{(x,y)/x=2y, x\in\{1,2,3,4\}, y\in\{1,2,3,4\}\})$ (Represent the given relation by (a) and arrow diagram (b) a graph (c) a set in roster form (Exercise 1.2 – 4(1))

Solution:

$\{(x,y)/x=2y, x\in\{1,2,3,4\}, y\in\{1,2,3,4\}\})$

$y=1\Rightarrow x=2(1)=2$

$y=2\Rightarrow x=2(2)=4$

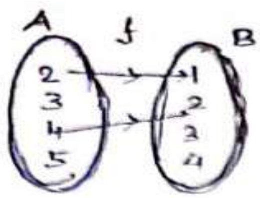
$y=3\Rightarrow x=2(3)=6$

$y=4\Rightarrow x=2(4)=8$

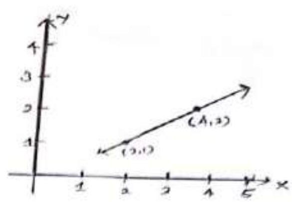
Roaster form

R = {(2,1),(4,2)}

Arrow diagram



Graph



5. $\{(x,y)/y=x+3, x,y$ are natural numbers <10}

(i) an arrow diagram (ii) a graph

(iii) a set in roaster form. (exercise 1.2 – 4 (ii))

Soluation:

x= {1,2,3,5,6,7,8,9}

$x=1\Rightarrow y=1+3=4$

$x=2\Rightarrow y=2+3=5$

$x=3\Rightarrow y=3+3=6$

$x=4\Rightarrow y=4+3=7$

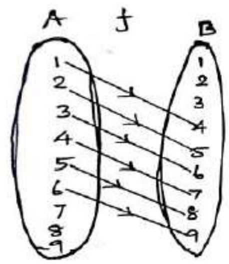
$x=5\Rightarrow y=5+3=8$

$x=6\Rightarrow y=6+3=9$

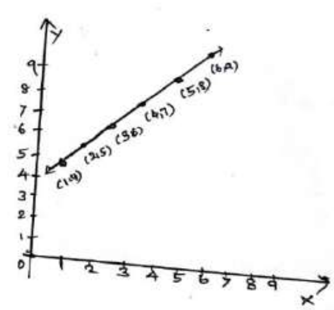
Roaster form

R= {(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)}

Arrow diagram



Graph



6. A={1,2,3,4}, B={2,5,8,11,14} be two sets $f:A\to B$ be a function given by f(x)=3x-1 Represent this function. (i) by arrow diagram (ii) in a table form (iii) as a set of ordered pairs (iv) in a graphical. (Example 1.11)

$f(x)=3x-1 \quad A=\{1,2,3,4\}$

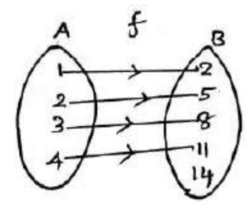
$x=1\Rightarrow f(1)=3(1)-1=3-1=2$

$x=2\Rightarrow f(2)=3(2)-1=6-1=5$

$x=3\Rightarrow f(3)=3(3)-1=9-1=8$

$x=4\Rightarrow f(4)=3(4)-1=12-1=11$

(i)Arrow diagram



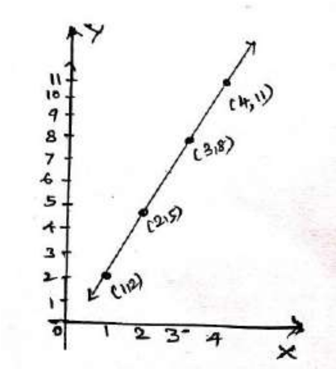
(ii)Table form

X	1	2	3	4
Y	2	5	8	11

(iii)Set of ordered pairs

$f = \{(1,2),(2,5),(3,8),(4,11)\}$

(iv)Graphical form



For Practice:

7.A company has four categories of employees given by assistants (A) clerks (c), managers (m) and an executive officer (E). The company provide Rs. 10,000, Rs. 25,000, Rs. 50,000 and Rs. 1,00,000 as salaries to the people who work in the categories A,C,M and E respectively. If A1, A2,A3, A4 were assistants: E1, E2 were clerks: M1,M2,M3 were managers and E1,E2 were executive officers and if the relation R is defined xRy, where x is the salary given to person of, express the relation R through an ordered pair and an arrow diagram. (Exercise 1.2 – 5)

8.Let $f : A \to B$ be a function defined by $f(x) = \frac{x}{2} - 1$ where A={2,4,6,10,12} B={0,1,2,4,5,9} Represent of ordered (i)pairs (ii) a table (iii) an arrow diagram (iv) a graph. (exercise 1.4 -2)

Solution:

$$f(x) = \frac{x}{2} - 1 \quad ; \quad A = \{2,4,6,10,12\}$$

$$x = 2 \Rightarrow f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$$

$$x = 4 \Rightarrow f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$$

$$x = 6 \Rightarrow f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$$

$$x = 10 \Rightarrow f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$$

$$x = 12 \Rightarrow f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$$

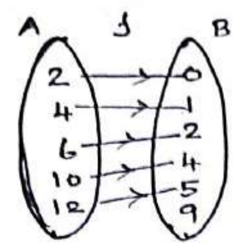
(i)A set of ordered pairs

$f = \{(2,0),(4,1),(6,.2),(10,4),(12,5)\}$

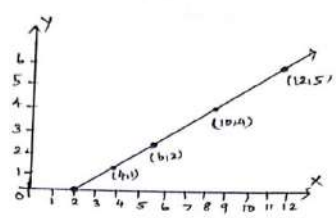
(ii)A table

X	2	4	6	10	12
Y	0	1	2	4	5

(iii) An arrow diagram



(iv) A graph



For practice

9..If $x = \{-5,1,3,4\}, y = \{a,b,c\}$ then which of the following relations are functions from x to y?

$i) R1 = \{(-5,a),(1,a),(3,b)\}$

$ii) R2 = \{(-5,b),(1,b),(3,a),(4,c)\}$

$iii) R3 = \{(-5,a),(1,a),(3,b),(4,c),(1,b)\}$

(Example.1.8)

10.If the function f is a defined by

$$f(x)=\begin{cases} x+2, & x>1.... \\ 2, & -1\le x<1 \\ x-1, & -3<x<-1 \end{cases}$$

find the values $(i)\ f(3)$ (ii) f(0) (iii) f(-1.5)

$(iv)\ f(2)+f(-2)$. (exercise 1.4-9)

Solution:

$$f(x)=\begin{cases} x+2, & x=2,3,4 \\ 2, & x=-1,0 \\ x-1, & x=-2,-1,1,1 \end{cases}$$

$$\text{(i) } f(3)=(x+2)$$
$$=(3+2)$$
$$=5$$

$$\text{(ii) } f(0)=2$$
$$=2$$

$$\text{(iii) } f\{-1.5\}=(x-1)$$
$$=(-1.5-1)=-2.5$$

$$\text{(iv) } f(2)+f(-2)=(x+2)+(x+1)$$
$$=(2+2)+(-2-1)$$
$$=4-3$$
$$=1$$

11..A Fuction $f:[-5,9]$ is

$$f(x)=\begin{cases} 6x+1, & -5\le x<2 \\ 5x^2-1, & 2\le x<6 \\ 3x-4 & 6\le x\le 9 \end{cases}$$

find $(i) f(-3)+f(2)\ (ii) f(7)-f(1)\ \ (iii)\ 2f(4)+f(8)$

$(iv)\ \dfrac{2f(-2)-f(6)}{f(4)+f(-2)}$ (Exercise 1.4-10)

Solution:

$$\begin{cases} 6x+1, & x=-5,-4,.-3,-2,-1,0,1 \\ 5x^2-1, & x=2,3,4,5 \\ 3x-4, & x=6,7,8,9 \end{cases}$$

$$\text{(i) } f(-3)+f(2)=(6x+1)\ x=-3+(5x^2-1)x=2$$
$$=(6(-3)+1)+(5(2)(2)-1)$$
$$=(-18+1)+(20-1)$$
$$=(-17)+(20-1)$$
$$=-17+19$$
$$=2$$

$$\text{(ii) } f(7)-f(1)=(3x-4)x=7-(6x+1)x=1$$
$$=(3(7)-4)-(6(1)+1)$$
$$=(21-4)-(6+1)$$
$$=17-7$$
$$=10$$

$$\text{(iii) } 2f(4)+f(8)=2(5x2-1)+(3x-4)$$
$$=2(5(4)(4)-1)+(3(8)-4)$$
$$=2(80-1)+(24-4)$$
$$=2(79)+(20)$$
$$=158+20=178.$$

$$NR=2f(-2)-f(6)$$
$$=2(6x+1)-(3x-4)$$
$$=2(6(-2)+1)-(3(6)-4)$$
$$=2(-12+1)-(18-4)$$
$$=2(-11)-(14)$$
$$=-22-14=-36$$

$$DR=f(4)+f(-2)$$
$$=(5x2-1)+(6x+1)$$
$$=(5(4)(4)-1)+(6(-2)+1)$$
$$=(80-1)+(-12+1)=79-11=68$$

$$\therefore \frac{f(-2)-f(6)}{f(4)+f(-2)}=\frac{-36}{68}=\frac{-9}{17}$$

12.If the function $f:\mathbb{R}\longrightarrow\mathbb{R}$ is defined by

$$f(x)=\begin{cases} 2x+7; & x<-2 \\ x^2-2; & -2\le x<3 \\ 3x-2; & x\ge 3 \end{cases}$$ then find the values of (i)

f(4) (ii) f(-2) (iii) f(4)+2f(1) iv) $\dfrac{f(1)-3f(4)}{f(-3)}$ (Example 1-18)

13.The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) a,b are constant.

i)Check if this relation is a function

ii) Find a and b.

iii) Find the height of a person whose forehand length is 40cm

iv) Find the length of a person if the height is 53.3 inches.

Length 'x' of forehand (in cm)	Height 'y' in inches.
35	56
45	65
50	69.5
55	74

Solution

i) $R=\{(35,56),(45,65),(50,79.5),(55,74)\}$

R is function.

ii) $x=35 \Rightarrow y=56$

$y=ax+b$

$56=35a+b$.......(1)

$x=45 \Rightarrow y=65$

$65=45a+b$......(2)

$45a+b=65$..........(1)

$35a+b=56$..........(2)

$10a = 9$

$a = \frac{9}{10}=0.9$

$a=0.9 \Rightarrow$

$56=35\times 0.9+b$

$56=31.5+b$

$b=56-31.5$

$b=24.5$

(iii) $x=40 \Rightarrow y=?$

$y=ax+b$

$=0.9(40)+24.5$

$=36+24.5$

=60.5 in ches.

iv) $y=53.3 \Rightarrow x=?$

$y=ax+b$

$53.3=0.9x+24.5$

$0.9x=53.3-24.5$

$=28.8$

$x=\frac{28.8}{0.9}$

$x=\frac{288}{9}$

$x = 32$ cm

14.The function 't' which maps temperature in Celsius (c) into temperature in Fahrenheit (F) is defined by t(c) = F where $F=\frac{9}{5}c+32)$

(i) t(0) (ii) t(28) (iii) t(-10)

iv) the value of C when t(c)=212

(v) the temperature when the Celsius value is equal to the Fahrenheit value

(Exercise 1.4 -12)

Solution:

t(c) = F

$\therefore t(c)=\frac{9c}{5}+32$

$t(0) = \frac{9(0)}{5} + 32$

$= 0 + 32$

$= 32^\circ F$

(ii) $t(28) = \frac{9(28)}{5} + 32$

$= \frac{252}{5} + 32$

$= 50.4 + 32$

$= 82.4^\circ F$

(iii) $t(-10) = \frac{9(-10)}{5} + 32$

$= \frac{-90}{5} + 32$

$= -18 + 32$

$= 14^\circ F$

(iv) $t(c) = 212,$

$212 = \frac{9c}{5} + 32$

$212 - 32 = \frac{9c}{5}.$

$180 = \frac{9c}{5}$

$9c = 180 \times 5$

$9c = 900, \quad c = \frac{900}{9}$

$\therefore c = 100^\circ c$

Celsius Value = Fahrenheit value.

$c = \frac{9c}{5} + 32$

$c = \frac{9c + 160}{5}$

$5c = 9c + 160 \Rightarrow 5c - 9c = 160$

$-4c = 160 \Rightarrow 4c = -160$

$c = \frac{-160}{4} \Rightarrow c = -40$

15. If $f(x) = x - 4, g(x) = x^2, h(x) = 3x - 5$ **Prove that** $(f \ 0 \ g) oh = f \ o \ (g \ o \ h)$.(**Exercise** 1.5 – 8(**iii**))

Solution:

$f(x) = x - 4, g(x) = x^2 h(x) = 3x - 5$

$(f \ o \ g)\, x = f(g(x))$

$= f(x^2))$

$= x^2 - 4$

$(f \ o \ g)\, oh\, (x) = (f \ o \ g)(3x\text{-}5)$

$= (3x\text{-}5)^2 - 4$

$= (3x)^2 - 2(3x)(5) + (5)^2$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21 \ldots\ldots\ldots\ldots (1)$

$(goh)x = g(h(x))$

$= g(3x\text{-}5)$

$= (3x\text{-}5)^2$

$= (3x)^2 - 2(3x)(5) + (5)^2$

$= 9x^2 - 30x + 25$

$fo(goh) = f(9x^2 - 30x + 25)$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21 \ldots\ldots\ldots\ldots (2)$

$(1) = (2)$

$(f \ o \ g) oh = f \ o \ (g \ o \ h)$

If $f(x) = 2x + 3, g(x) = 1 - 2x, h(x) = 3x$ **prove that** $(fog)\, oh = f \ o \ (g \ o \ h)$.

$(f \ o \ g)\, x = f \ o \ g(x)$

$= f(1\text{-}2x)$

$= 2(1\text{-}2x) + 3$

$= 2\text{-}4x + 3$

$= 5\text{-}4x$

$(f \ o \ g)\, oh\, (x) = (f \ o \ g)(3x)$

$= 5 - 4(3x)$

$= 5\text{-}12x \ldots\ldots\ldots (1)$

$(g\ .\ h) \times = g(h(x))$
$= g(3x)$
$= 1\text{-}2(3x)$
$= 1\text{-}6x$

$f\ o\ (g\ o\ h)x = f(1\text{-}6x)$
$= 2(1\text{-}6x)+3$
$= 2 - 12x+3$
$=5\text{-}12x.......(2)$

(1) = (2)

$(f\ o\ g)\ oh = f\ o\ (g\ o\ h)$

2. **Find x if g f f(x) = f gg(x), given $f(x) = 3x + 1, g(x) = x + 3$ (Example .1.24)**

Solution

$gf\ f(x) = g\{f[f(x)]\}$
$= g\{f\{3x+1]+1\}$
$= g(9x+3+1]\}$
$= g\{9x+4\}$
$= 9x+4+3$
$= 2x+7..............(1)$

$f\ g\ g(x) = f\ \{g\{g(x)]\}$
$= f\{g[x+3]\}$
$= f\{(x+3)+3\}$
$= f\{x+3+3\}$
$= f\{x+6)$
$= 3(x+6)+1$
$= 3x+18+1$
$= 3x+19.......(2)$

$(1) = (2)$
$9x+7 = 3x+129$
$9x - 3x = 19 - 7$
$6x = 12,$

$x = \frac{12}{6} = 2$
$\therefore x = 2$

For practice

18. **Consider the functions $f(x), g(x), h(x)$ as given below, show that $(f\ o\ g)\ oh = f\ o\ (g\ o\ h)$ in each case. (exercise 1.5-8)**

(i) $f(x) = x - 1, g(x) = 3x + 1, h(x) = x^2$
(ii) $f(x) = x^2, g(x) = 2x, h(x) = x + 4$

2. NUMBERS AND SEQUENCES

FORMULAS

ARITHMETIC PROGRESSION

1) n^{th} term $t_n = a + (n-1)d$
2) $d = t_2 - t_1$
3) If the given terms are in A.P, $t_2 - t_1 = t_3 - t_2$
4) $n = \frac{l-a}{d} + 1$
5) Sum to first n terms , $S_n = \frac{n}{2}[2a + (n-1)d]$
6) If the last term l is given, then $S_n = \frac{n}{2}(a + l)$

Special Series

7) The sum of first n natural numbers $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$
8) The sum of squares of first n natural numbers

$$1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

9) The sum of cubes of first n natural numbers

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

10) The sum of first n odd natural numbers

$$1 + 3 + 5 + \ldots + (2n-1) = n^2$$

TWO MARK QUESTIONS

1) A Man has 532 flower pots. He wants to arrange them in rows such that each rows contains 21 flower pots. Find the number of completed rows and how many flower pots are left over. **EX:2.1(2)**

Solution

No. of flower pots = 532

All pots to be arranged in rows & each row to contain 21 flower pots.

$\therefore 532 = 21q + r$

$532 = 21 \times 25 + 7$

$\therefore$Number of completed rows = 25

Number of flower pots left out = 7

---

2) Is 7 x 5 x 3 x 2 + 3 a composite number ? Justify your answer. **Eg:2.9**

Solution

7 x 5 x 3 x 2 +3

= 3 x (7 x 5 x 2 + 1)

= 3 x 71

Since the given number can be factorized in terms of two primes, it is a composite number.

---

3) 'a ' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a ' and ' b' . **Eg: 2.10**

Solution

$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 2^5 \times 5^2$

Hence, $a^b \times b^a = 2^5 \times 5^2$

$\therefore$ a = 2 and b = 5 (or) a = 5 and b =2.

4) Find the HCF of 252525 and 363636
EX:2.2(3)

Solution

5	252525
5	50505
3	10101
7	3367
	481

2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
	481

$$252525 = 5 \times 5 \times \underline{3} \times \underline{7} \times \underline{481}$$

$$363636 = 2 \times 2 \times \underline{3} \times 3 \times 3 \times \underline{7} \times \underline{481}$$

$$= 3 \times 7 \times 481$$

$$= 10101$$

5) If $13824 = 2^a \times 3^b$ then find a and b. **EX:2.2(4)**

Solution

$13824 = 2^a \times 3^b = 2^9 \times 3^3$

$\therefore$ a = 9 and b = 3

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

6) If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4. **EX: 2.2 (5)**

Solution

$113400 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7$

$x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1$

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
	7

7) Find the least number that is divisible by the first ten natural numbers. **EX: 2.2 (9)**

Solution

First ten natural numbers are 1,2,3,4,5,6,7,8,9,10.

Find LCM

2 = 2 x 1

3 = 3 x 1

4 = 2 x 2

5 = 5 x 1

6 = 2 x 3

7 = 7 x 1

8 = 2 x 2 x 2

9 = 3 x 3

10 = 2 x 5

LCM = 2 X 2 X 2 X 3 X 3 X 5 X 7

The Least number = 2520

8) Find the next three terms of the sequences. **Eg:2.19**

(i) 1/2 , 1/6 , 1/10 , 1/14,.......

Solution

In the above sequence the denominator is increased by 4.

So the next three terms are

a_5 = 1/14 + 4 = 1/18

a_6 = 1/18 + 4 = 1/22

a_7 = 1/22 + 4 = 1 /26

So the next three terms are 1/18, 1/22 ,1/26

9) Find the next three terms
8, 24 , 72 .... **EX: 2.4 (1(i))**

8 x 3 = 24

24 x 3 = 72

So, the next three terms are

72 x 3 = 216

216 x 3 = 648

648 x 3 = 1944

For Practice

Find the next three terms

10) 5,2,-1,-4,.... **Eg:2.9**

11) 1,0.1,0.01,....

12) 5 ,1 , -3,.... **Ex: 2.4**

13) 1/4 , 2/9, 3/16,....

14) Find the general term for the following sequences. **Eg: 2.20**

(i) 3,6,9,....

Solution

Here the terms are multiple of 3. So, the general term is a_n = 3n.

15) Find the first four terms of the sequences whose n^{th} terms are given by

$a_n = n^3 - 2$ **EX: 2.4(2)**

Solution

$a_1 = 1^3 - 2 = 1 - 2 = -1$

$a_2 = 2^3 - 2 = 8 - 2 = 6$

$a_3 = 3^3 - 2 = 27 - 2 = 25$

$a_4 = 4^3 - 2 = 64 - 2 = 62$

For Practice

Find the nth term

16) 1/2, 2/3, 3/4 ....**Eg:2.20**

17) 5,-25,125,....

Find the first four terms

18) $a_n = (-1)^{n+1} n(n+1)$ **EX: 2.4 (2)**

19) $a_n = 2n^2 - 6$

20) Find the nth term of the following sequences.**EX: 2.4 (3)**

(i) 2,5,10,17....

(ii) 0,1/2, 2/3,... (iii) 3,8,13,18,....

21) Find the indicated terms **EX: 2.4(4)**

$a_n = \frac{5n}{n+2}$; a_6 and a_{13}

$a_6 = \frac{5 X 6}{6+2} = \frac{30}{8} = \frac{15}{4}$

$a_{13} = \frac{5 X 13}{13+2} = \frac{65}{15} = \frac{13}{3}$

For Practice

22) Find the indicated terms

$a_n = -(n^2 - 4)$;a_4 and a_{11} **EX: 2.4 (4)**

23) The general term of a sequence is defined as

$a_n =$ n(n+3); n Є N is odd
n^2 + 1; n Є N is even **Eg: 2.21**

Find the eleventh and eighteenth terms.

Solution

$a_{11} = 11(11 + 3)$

$= 11 \times 14$

$= 154$

$a_{18} = 18^2 + 1$

$= 324 + 1 = 325$

For Practice

24) Find a_8 and a_{15} whose n^{th} term is

$a_n = \frac{n^2-1}{n+3}$; n is even, n Є N

$\frac{n^2}{2n+1}$; n is odd, n Є N **EX: 2.4 (5)**

25) If $a_1 = 1$, $a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$, n≥ 3, Є N, then find the first six terms of the sequence. **EX: 2.4 (6)**

Solution

$a_1 = 1$, $a_2 = 1$

$a_n = 2a_{n-1} + a_{n-2}$

$a_3 = 2a_{3-1} + a_{3-2}$

$= 2a_2 + a_1$

$= 2 \times 1 + 1$

$= 2 + 1 = 3$

$a_4 = 2a_{4-1} + a_{4-2}$

$= 2a_3 + a_2$

$= 2 \times 3 + 1$

$= 6 + 1 = 7$

$a_5 = 2a_{5-1} + a_{5-2}$

$= 2a_4 + a_3$

$= 2 \times 7 + 3$

$= 14 + 3 = 17$

$a_6 = 2a_{6-1} + a_{6-2}$

$= 2a_5 + a_4$

$= 2 \times 17 + 7$

$= 34 + 7 = 41$

The First six terms are

1,1,3,7,17,41

For Practice

26) Find the first five terms of the following sequence. **EX: 2.22**

$a_1 = 1$, $a_2 = 1$, $a_n = \frac{a_{n-1}}{a_{n-2} + 3}$; n≥ 3, n Є N

27) Check whether the following sequences are in A.P. or not ? **Eg:2.23**

(i) X + 2 , 2x + 3, 3x + 4…….

Solution

$t_2 - t_1 = (2x + 3) - (X + 2)$

$= 2x + 3 - x - 2$

$= x + 1$

$t_3 - t_2 = (3x + 4) - (2x + 3)$

$= 3x + 4 - 2x - 3$

$= x + 1$

$t_2 - t_1 = t_3 - t_2$

Hence the sequence X + 2 , 2x + 3, 3x + 4……. is in A.P.

28) Check a -3 , a -5, a -7,…..are in A.P. **EX: 2.5 (1-i)**

Solution

$t_2 - t_1 = a - 5 - (a - 3)$

$= a - 5 - a + 3$

$= -2$

$t_3 - t_2 = a - 7 - (a - 5)$

$= a - 7 - a + 5$

$= -2$

$t_2 - t_1 = t_3 - t_2$

Hence the sequence a -3, a - 5, a – 7….. is in A.P.

For Practice

Check whether the following sequences are in A.P.

29) 2, 4,8,16,……. **Eg:2.23(ii)**

30) ½ , 1/3, ¼, 1/5,… **EX: 2.5**
31) 9,13,17, 21,25..
32) -1/3,0,1/3,2/3,….
33) 1,-1,1,-1,1,-1….

34) Write an A.P. whose first term is 20 and common difference is 8. **Eg:2.24**

Solution

a = 20

d = 8

Arithmetic Progression is a , a+d , a +2d, a+ 3d ,……

20, 20 + 8, 20 + 2(8), 20 + 3(8)……

So, A.P. is 20 , 28 , 36 , 44……..

For Practice

35) a = 5 , d = 6 **EX: 2.5(2)**

36) a =7, d = -5

37) a = ¾ , d = ½

38) Find the number of terms in the A.P.

3, 6, 9, 12,……111 **Eg: 2.26**

Solution

a = 3

d = 6 -3 = 3

last term l = 111

$n = \left[\frac{l-a}{d} + 1\right]$

$n = \left[\frac{111-3}{3} + 1\right]$

n = 108/3 +1

n = 36 + 1 = 37

Thus the A.P. contain 37 terms.

39) Find the 19th term of an A.P. -11 , -15 , -19, ……. **EX: 2.5(4)**

Solution

a = -11

d = -15 – (-11) = -15 + 11 = -4

t_n = a + (n – 1)d

= -11 + (19 – 1) -4

= -11 + 18 x -4

= -11-72

= - 83

40) Which term of an A.P. **EX: 2.5(5)**

16 , 11, 6, 1, ……. is -54?

solution

a = 16

d = 11 -16 = -5

Find n

t_n = a + (n -1)d

-54 = 16 + (n – 1) -5

-54 = 16 +(-5n) + 5

5n = 54 + 21

5n = 75

n = 75/5

n = 15

41) If 3 + k , 18 – k , 5k + 1 are in A.P. Then find k. **EX:2.5(8)**

Solution

$t_2 - t_1 = t_3 - t_2$

18 –k –(3 + k)= 5k +1 – (18 – k)

18 – k – 3 –k = 5k + 1 – 18 + k

$15 - 2k = 6k - 17$

$6k + 2k = 15 + 17$

$8k = 32$

$k = 32/8$

$k = 4$

For Practice

42) Find x, y and z given that the numbers x, 10, y, 24, z are in A.P. **EX:2.5(9)**

43) Find the sum of first 15 terms of the A.P. **Eg:2.31**

8, 7 ¼ , 6 ½, 5 ¾ ,….

$a = 8$

$d = 7\frac{1}{4} - 8$

$= \frac{29}{4} - 8$

$= \frac{29-32}{4} = \frac{-3}{4}$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{15} = \frac{15}{2}\left[2\times 8 + (15-1)(-\frac{3}{4})\right]$

$S_{15} = \frac{15}{2}\left[16 - \frac{21}{2}\right] = \frac{165}{4}$

For Practice

Find the sum of the following. **EX:2.6**

44) 3, 7,11,….up to 40 terms.

45) 102, 97,92,…up to 27 terms.

46) 6 + 13 + 20 +…+ 97

47)Find the sum of the following series

1 + 2 + 3 +….+ 60 **EX: 2.9**

Solution

$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

$1 + 2 + 3 + \ldots\ldots + 60 = \frac{60(60+1)}{2}$

$= \frac{60 \times 61}{2}$

$= 30 \times 61 = 1830$

48) 3 + 6 + 9 +…..+ 96

Solution

$3(1 + 2 + 3 + \ldots\ldots + 32)$

$= 3 \times \frac{32(32+1)}{2}$

$= 3 \times 16 \times 33$

$= 1584$

49) 51 + 52 + 53 +…+ 92

Solution

$1 + 2 + 3 + \ldots\ldots + 92 - (1 + 2 + 3 + \ldots\ldots + 50)$

$= \frac{92(92+1)}{2} - \frac{50(50+1)}{2}$

$= 46 \times 93 - 25 \times 51$

$= 4278 - 1275$

$= 3003$

50) 1 + 4 + 9 + 16 + ……+ 225

Solution

$1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 15^2$

$1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + n^2$

$= \frac{n(n+1)(2n+1)}{6}$

$= \frac{15(15+1)(2 \times 15+1)}{6}$

$= \frac{15(15+1)(2 \times 15+1)}{6}$

$= \frac{15 \times 16 \times (30+1)}{6}$

$= 5 \times 8 \times 31$

$= 1240$

51) $10^3 + 11^3 + 12^3 + ... + 20^3$

Solution

$(1^3 + 2^3 + 3^3 + ... + 20^3) - (1^3 + 2^3 + 3^3 + ... + 9^3)$

$= \left(\frac{n(n+1)}{2}\right)^2 - \left(\frac{n(n+1)}{2}\right)^2$

$= \left(\frac{20(20+1)}{2}\right)^2 - \left(\frac{9(9+1)}{2}\right)^2$

$= \left(\frac{20 X 21}{2}\right)^2 - \left(\frac{9 X 10}{2}\right)^2$

$= (210)^2 - (45)^2$

$= 44100 - 2025$

$= 42075$

For Practice

Find the sum

52) $6^2 + 7^2 + 8^2 + ...+ 21^2$ **EX: 2.9**

53) 1 + 3 + 5 + ... + 71

54) 1 + 2 + 3 + ...+ 50 **Eg:2.54**

55) 16 + 17 + 18 + ...75

56) 1 + 3 + 5 + ... + 40 terms **Eg:2.55**

57) 2 + 4 + 6 + ...+ 80

58) 1 + 3 + 5 + ... + 55

59) $1^2 + 2^2 + 3^2 + ...+ 19^2$ **Eg:2.56**

60) $5^2 + 10^2 + 15^2 + ...+ 105^2$

61) $15^2 + 16^2 + 17^2 + ...+ 28^2$

62) $1^3 + 2^3 + 3^3 + ...+ 16^3$ **Eg:2.57**

63) $9^3 + 10^3 + ...+ 21^3$

64) If 1 + 2 + 3 + ... +n = 666 then find n. **EX : 2.58**

Solution

$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

$\frac{n(n+1)}{2} = 666$

$n^2 + n = 1332$, $n^2 + n - 1332 = 0$

$(n + 37)(n - 36) = 0$

n = -37 or n = 36

But $n \neq -37$,Hence n = 36.

65) If If 1 + 2 + 3 + ...+ k = 325, then find $1^3 + 2^3 + 3^3 + ... + k^3$ **EX: 2.9**

$1^3 + 2^3 + 3^3 + ... + k^3 = \left(\frac{k(k+1)}{2}\right)^2$

Given, 1 + 2 + 3 + ...+ k = 325

$\frac{k(k+1)}{2} = 325$

$\left(\frac{k(k+1)}{2}\right)^2 = 325^2$

$1^3 + 2^3 + 3^3 + = ... + k^3 = 105625$

For Practice

66) If $1^3 + 2^3 + 3^3 + ... + k^3 = 44100$ then find 1 + 2 + 3 + ...+ k. **EX: 2.9(3)**

67) Rekha has 15 square colour papers of sizes 10cm,11cm, 12cm, ...24cm. Howmuch area can be decorated with these colour papers? **EX: 2.9(6)**

FIVE MARKS QUESTIONS

1) Find the HCF of 396, 504, 636. **Eg:2.6**

Solution

Find HCF of 396 , 504

Using Euclid's division algorithm, We get

504 = 396 x 1 + 108 , $108 \neq 0$

396 = 108 x 3 + 72 , $72 \neq 0$

108 = 72 x 1 + 36 , $36 \neq 0$

72 = 36 x 2 + 0

HCF of 396 , 504 = 36

Find HCF of 636 , 36

636 = 36 x 17 + 24 , 24 ≠ 0

36 = 24 x 1 + 12, 12 ≠ 0

24 = 12 x 2 + 0

HCF of 636 , 36 = 12

∴ HCF of 396, 504, 636 = 12

2) 340 and 412 **EX: 2.1(6)**

Using Euclid's division algorithm, We get

412 = 340 x 1 + 72 , 72 ≠ 0

340 = 72 x 4 + 52, 52 ≠ 0

72 = 52 x 1 + 20 , 20 ≠ 0

52 = 20 x 2 + 12 , 12 ≠ 0

20 = 12 x 1 + 8, 8 ≠ 0

12 = 8 x 1 + 4 , 4 ≠ 0

8 = 4 x 2 + 0

∴ HCF of 340, 412 = 4

For Practice

Find HCF of 3) 867 and 255

4) 10224 and 9648

5) 84, 90 and 120

6) Determine the general term of an A.P. whose 7^{th} term is -1 and 16^{th} term is 17 .**Eg : 2.27**

Solution

$t_7 = -1$, $t_{16} = 17$

$t_n = a + (n-1) d$

a + (7-1) d = -1

a + 6d = -1 ⟶ 1

a + (16 -1) d = 17

a + 15d = 17 ⟶ 2

subtract 1 from 2 , we get

9d = 17 –(-1)

9d = 17+ 1 = 18

d = 18/9 = 2

Sub d = 2 in 1 , a + 6 x 2 = -1

a + 12 = -1

a = -1 -12

a = -13

Hence , general term

$t_n = a + (n -1) d$

$t_n = -13 + (n -1) 2$

= -13 + 2n -2

= 2n – 15

7) In an A.P., sum of four consecutive terms is 28 and the sum of their squares is 276. Find the four numbers. **Eg: 2.29**

Solution

Let us take the four terms in the form (a-3d) , (a – d), (a +d) and (a + 3d).

Sum of the four terms is 28.

a-3d + a – d + a +d + a + 3d = 28

4a = 28

a = 28 /4

a = 7

sum of their squares is 276

$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276.$

$a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 276$

$4a^2 + 20d^2 = 276$

$4 \times 7^2 + 20d^2 = 276$

$4 \times 49 + 20d^2 = 276$

$20d^2 = 276 - 196$

$20d^2 = 80$

$d^2 = 80/20$

$d^2 = 4$

$d = \pm 2$

The four numbers are 7- 3(2), 7-2, 7 +2, 7 +3(2)

∴ 1, 5, 9 and 13.

8) Find the middle term(s) of an A.P. 9, 15, 21, 27,.....183. **EX:2.5 (6)**

Solution

a = 9 , d = 15 – 9 = 6 , l = 183

$$n = \frac{l-a}{d} + 1$$
$$= \frac{183 - 9}{6} + 1$$
$$= \frac{174}{6} + 1$$
$$= 29 + 1$$
$$n = 30$$

middle terms are t_{15} and t_{16}

$$\mathbf{t_n = a + (n-1)\,d}$$
$$t_{15} = 9 + (15 - 1) \times 6$$
$$= 9 + 14 \times 6$$
$$= 9 + 84$$
$$= 93$$
$$t_{16} = 9 + (16 - 1) \times 6$$
$$= 9 + 15 \times 6$$
$$= 9 + 90$$
$$= 99$$

Middle terms are 93 , 99.

9) If nine times ninth term is equal to the fifteenth term, show that six times twenty fourth term is zero. **EX:2.5 (7)**

Solution

Given, $9\ t_9 = 15\ t_{15}$

To Prove : $6\ t_{24} = 0$

$9\ t_9 = 15\ t_{15}$

9 [a + (9 – 1) d] = 15 [a + (15 -1) d]

9 [a + 8d] = 15 [a + 14d]

9a + 72d = 15a + 210d

15a -9a + 210 d - 72 d = 0

6a +138 d = 0

6 (a + 23d) = 0

∴ $6\ t_{24} = 0$

For Practice EX:2.5(10,11)

10) In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

11) The sum of three consecutive terms that are in A.P is 27 and their product is 288. Find the three terms.

12) Find the sum of all natural numbers between 300 and 600 which are divisible by 7. **Eg:2.36**

Solution

301 + 308 + 315 + ...595

The term of the above series are in A.P.

a = 301 , d = 7 , l = 595

$$n = \frac{l-a}{d} + 1$$
$$= \frac{595 - 301}{7} + 1$$
$$= \frac{294}{7} + 1 = 42 + 1$$
$$n = 43$$

$$S_n = \frac{n}{2}(a + l)$$
$$= \frac{43}{2}(301 + 595)$$
$$= \frac{43}{2} \times 896 = 43 \times 448$$
$$= 19264$$

For Practice

13) Find the sum of all odd positive integers less than 450. **EX:2.6(6)**

14) Find the sum of all natural numbers between 602 and 902 which are not divisible by 4. **EX:2.6(7)**

15) A man repays a loan of ₹ 65, 000 by paying ₹ 400 in the first month and then increasing the payment by ₹ 300 every month. How long will it take for him to clear the loan? **EX:2.6(9)**

16) Find the sum to n terms of the series 5 + 55 + 555 + ….. **Eg:2.51**

Solution

5 + 55 + 555 + …..

= 5(1 + 11 + 111 +…. + n terms)

$= \frac{5}{9}$ (9 + 99 + 999 + …….. + n terms)

$= \frac{5}{9}$ ((10 -1) + (100 – 1) + (1000-1)+……nterms)

$= \frac{5}{9}$ [(10 + 100 + 1000 +……+ nterms) - n]

$$= \frac{5}{9}\left[\frac{10(10^n - 1)}{(10 - 1)} - n\right]$$

$$= \frac{50(10^n - 1)}{81} - \frac{5n}{9}$$

17) 3 + 33 + 333 + ………to n terms

EX: 2.8(6)

Solution

3 + 33 + 333 + …..

= 3 (1 + 11 + 111 +…. + n terms)

$= \frac{3}{9}$ (9 + 99 + 999 + …….. + n terms)

$= \frac{3}{9}$ ((10 -1) + (100 – 1) + (1000-1)+……+ nterms)

$= \frac{3}{9}$ [(10 + 100 + 1000 +…+ nterms) - n]

$$= \frac{3}{9}\left[\frac{10(10^n - 1)}{(10 - 1)} - n\right]$$

$$= \frac{30}{81}(10^n - 1) - \frac{3n}{9}$$

For Practice

18) 0.4 + 0.44 + 0.444 + …. to n terms

UNIT-III. ALGEBRA

FORMULAS

1. The roots of the quadratic equation are given by $\pm\sqrt{b}$
2. Sum of the roots
3. Product of the roots
4. If the roots of a quadratic equation are α and , then the equation is given by
5. The value of the discriminant decides the nature of roots as follows
   (i) When $\Delta > 0$, the roots are real and unequal
   (ii) When the roots are real and equal
   (iii) When there are no real roots.
6.
7.
8.
9.
10.
11.
12. (

TWO MARKS

1. **Solve: = 6,** *(Example - 3.2)*
   Soln.:

   subsituting

   we get,

   Therefore.

2. **Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?** *(Ex - 3.6 - 7)*

   Soln: Pari's work in one hour = part

   Yuvan's work in one hour = part

   Pari and Yuvan's work in one hour = +

   = = part

   To complete part, it takes one hour.

   Hence, it takes hours to complete the whole work. i.e. 2 hours and 24 minutes.

3. **Simplify:** *(Ex - 3.5 – 1(i))*
   Soln.:

   =

4. **Simplify:** *(Ex - 3.6 – 1(iii))*
   Soln.:

$$\frac{x^3}{x-y}+\frac{y^3}{y-x}$$
$$=\frac{x^3}{x-y}-\frac{y^3}{x-y}$$
$$=\frac{x^3-y^3}{x-y}$$
$$=\frac{(x-y)(x^2+xy+y^2)}{x-y}$$
$$=x^2+xy+y^2$$

5. **Find the square root of**
   (Example - 3.19 – (ii))
   Soln.:

FOR PRACTICE

(i) 8 4 16 20

(ii) ________________

6. **Find the zeros of the quadratic expression $x^2+8x+12$** *(Example - 3.23)*

Soln.:

Let $p(x) = x^2 + 8x + 12 = (x + 2)(x + 6)$

$p(-2) = 4 - 16 + 12 = 0$

$p(-6) = 36 - 48 + 12 = 0$

Therefore -2 and -6 are zeros of $p(x)=x^2+8x+12$

7. **Write down the quadratic equation in general form for which sum and product of the roots are given below. — , –** *(Example - 3.24-(ii))*

Soln.: General form of the quadratic equation when the roots are given is

$$x^2 - \left(-\frac{7}{2}\right)x\,,\frac{5}{2} = 0 \text{ gives } 2x^2 + 7x + 5 = 0$$

x^2 – (S.O.R) x + P.O.R = 0

FOR PRACTICE..

(i) 9, 14 (ii) 9, 20 (iii) — , —

8. **Find the sum and product of the roots for each of the following quadratic equation $x^2 + 8x - 65 = 0$** *(Example - 3.25)*

Soln.:

$x^2 + 8x - 65 = 0$

$a = 1, b = 8, c = -65$

$$\alpha + \beta = -\frac{b}{a} = -8 \text{ and } \alpha\beta = \frac{c}{a} = -65$$

$\alpha + \beta = -8$; $\alpha\beta = -65$

9. **Solve : $2x^2 - 3x - 3 = 0$ by formula method** *(Example - 3.33)*

Soln.:

Compare $2x^2 - 3x - 3 = 0$ with the standard form $ax^2 + bx + c = 0$

$a = 2, b = -3, c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

substituting the values of a, b and c in the formula we get,

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)} = \frac{3 \pm \sqrt{33}}{4}$$

Therefore, $x = \dfrac{3+\sqrt{33}}{4}$, $x = \dfrac{3-\sqrt{33}}{4}$

10. **The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?** *(Example - 3.36)*

Soln.:

Let the present age of Kumaran be x years.

Two years ago, his age $= (x - 2)$ years.

Four years from now, his age $= (x + 4)$ years.

Given, $(x - 2)(x + 4) = 1 + 2x$

$x^2 + 2x - 8 = 1 + 2x$ gives $(x - 3)(x + 3) = 0$

then, $x = \pm 3$

Therefore, $x = 3$ (Rejecting -3 as age cannot be negative)

Kumaran's present age is 3 years.

FOR PRACTICE...

(i) If the difference between a number and its reciprocal is —, find the number.

(ii) A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.

11. **If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements** *(Ex - 3.17 - 2)*

Soln.

Given, a matrix has 18 elements

The possible orders of the matrix are

$18 \times 1,\ 1 \times 18,\ 9 \times 2,\ 2 \times 9,\ 6 \times 3,\ 3 \times 6$

If the matrix has 6 elements

The order are $1 \times 6,\ 6 \times 1,\ 3 \times 2,\ 2 \times 3$

12. **Construct a 3 × 3 matrix whose elements are given by $a_{ij} = |i - 2j|$** *(Ex - 3.17 – 3(i))*

Soln.

Given $a_{ij} = | i - 2j |$, 3×3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = | 1 - 2 | = | -1 | = 1$

$a_{12} = | 1 - 4 | = | -3 | = 3$

$a_{13} = | 1 - 6 | = | -5 | = 5$

$a_{21} = | 2 - 2 | = 0$

$a_{22} = | 2 - 4 | = | -2 | = 2$

$a_{23} = | 2 - 6 | = | -4 | = 4$

$a_{31} = | 3 - 2 | = | 1 | = 1$

$a_{32} = | 3 - 4 | = | -1 | = 1$

$a_{33} = | 3 - 6 | = | -3 | = 3$

$$\therefore\ A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$$

FOR PRACTICE...

Construct a 3 × 3 matrix whose elements are given by (i) $a_{ij} = \frac{(i+j)^3}{3}$ (ii) $a_{ij} = i^2 j^2$

13. **If $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A** *(Ex - 3.17 - 4)*

Soln. Given

$$A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$$

$$\therefore\ A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

FOR PRACTICE...

If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of $-A$

If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T = A$.

14. **Find the value of a, b, c, d from the equation $\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$** *(Example - 3.59)*

Soln.

$a - b = 1$

$2a - b = 0 \implies 2a = b$

$2a + c = 5 \qquad 3c + d = 2$

$a - 2a = 1 \qquad -a = 1 \qquad a = -1$

$-1 - b = 1 \qquad -b = 1 + 1 = 2 \qquad b = 2$

$2(-1) + c = 5$

$-2 + c = 5 \qquad c = 5 + 2 = 7$

$3 \times 7 + d = 2$

$21 + d = 2 \qquad d = 2 - 21 = -19$

FOR PRACTICE...

In the matrix $A = \begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$ Write

(i) The number of elements (ii) The order of the matrix (iii) Write the elements $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$.

15. If $A = \begin{pmatrix} & 9 \end{pmatrix}$ $B = \begin{pmatrix} -1 & & 4 \end{pmatrix}$

then find **B**. *(Example - 3.63*

Soln.

Since A and B have same order 3 × 3, 2A + B is defined.

$$\text{We have } 2A+3 = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

FIVE MARKS

1. **Solve the following system of linear equations in three variables.**

5; = 9;

(Ex - 3.1 – 1(i)

Soln.

Given $x + y + z = 5$ — (1)

$2x - y + z = 9$ — (2)

$x - 2y + 3z = 16$ — (3)

(1) – (3) ⇒ $3y - 2z = -11$ — (4)

(2) ⇒ $2x - y + z = 9$

(1) × 2 ⇒ $2x + 2y + 2z = 10$ (–)

Subtracting $-3y - z = -1$ — (5)

Solving (4) & (5)

$3y - 2z = -11$

$-3y - z = -1$

Adding $-3z = -12$

$z = 4$

Sub z = 4 in (5)

$-3y - 4 = -1$

⇒ $-3y = 3$

⇒ $y = -1$

sub y = – 1, z = 4 in (1)

⇒ $x - 1 + 4 = 5$

⇒ $x = 2$

∴ Solution set :

$x = 2, y = -1, z = 4$

2. **Vani, her father and her grandfather have an average age of 53. One-half of her grandfather's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago, if Vani's grand father was four times as old as Vani then how old are they all now?** *(Ex.3.1 – 3)*

Soln.

Let the present age of Vani, her father, grand father be x, y, z respectively.

By data given,

$\frac{x+y+z}{3} = 53 \Rightarrow x + y + z = 159$ .... (1)

$\frac{1}{2}z + \frac{1}{3}y + \frac{1}{4}x = 65$

⇒ $\frac{6z + 4y + 3x}{12} = 65 \Rightarrow$

$3x + 4y + 6z = 780$ .... (2)

$(z - 4) = 4(x - 4) \Rightarrow 4x - z = 12$ .... (3)

Solving (1) & (2)

(1) × (4) ⇒ $4x + 4y + 4z = 636$

(2) ⇒ $x + 4y + 6z = 780$

Subtracting $3x - 2z = -144$ .... (4)

Solving (3) & (4)

(3) × (2) ⇒ $8x - 2z = 24$

(4) ⇒ $3x - 2z = -144$

Subtracting $7x = 168$

$x = \frac{168}{7} = 24$

Sub x = 24 in (3)

$96 - z = 12$

$z = 84$

$\therefore$ (1) $\Rightarrow$ $24 + y + 84 = 159$

$\Rightarrow$ $y = 51$

$\therefore$ Vani' present age = 24 years

Father's present age = 51 years

Grand father's age = 84 years

3. **Find the GCD of the given polynomials $x^4 + 3x^3 - x - 3$, $x^3 + x^2 - 5x + 3$**
(Ex.3.2 – 1(i))
Soln.

Let $f(x) = x^4 + 3x^3 - x - 3$

$g(x) = x^3 + x^2 - 5x + 3$

To find the GCD of f(x), g(x)

Divide f(x) by g(x)

$$\begin{array}{r|l} & x + 2 \\ \hline x^3 + x^2 - 5x + 3 & x^4 + 3x^3 + 0x^2 - x - 3 \\ & x^4 + x^3 - 5x^2 + 3x \\ \hline & 2x^3 + 5x^2 - 4x - 3 \\ & 2x^3 + 2x^2 - 10x + 6 \\ \hline & 3x^2 + 6x - 9 \end{array}$$

$= 3(x^2 + 2x - 3) \neq 0$

Now , divide g(x) by $x^2 + 2x - 3$ (excluding 3)

$$\begin{array}{r|l} & x - 1 \\ \hline x^2 + 2x - 3 & x^3 + x^2 - 5x + 3 \\ & x^3 + 2x^2 - 3x \\ \hline & -x^2 - 2x + 3 \\ & -x^2 - 2x + 3 \\ \hline & 0 \end{array}$$

$\therefore$ Remainder becomes 0.

$\therefore$ The corresponding quotient is the HCF

$\therefore$ HCF $= x^2 + 2x - 3$

FOR PRACTICE *(Example* 3.10)
Find the GCD of the polynomials
$x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$

4. **Find the LCM of the each pair of the following polynomials $a^2 + 4a - 12$, $a^2 - 5a + 6$ whose GCD is $a - 2$** *(Ex.3.3 – 2(i))*
Soln:

Let f(x) $= a^2 + 4a - 12$

$= (a + 6)(a - 2)$

g(x) $= a^2 - 5a + 6$

$= (a - 3)(a - 2)$

GCD $= a - 2$

$\therefore$ LCM $= \dfrac{f(x) \times g(x)}{GCD}$

$= \dfrac{(a+6)(a-2) \times (a-3)(a-2)}{a-2}$

$= (a + 6)(a - 3)(a - 2)$

5. **Simplify:** $\dfrac{1}{x^2-5x+6} + \dfrac{1}{x^2-3x+2} - \dfrac{1}{x^2-8x+15}$ *(Example - 3.18)*

Soln.

$= \dfrac{1}{x^2-5x+6} + \dfrac{1}{x^2-3x+2} - \dfrac{1}{x^2-8x+15}$

$= \dfrac{1}{(x-2)(x-3)} + \dfrac{1}{(x-2)(x-1)} - \dfrac{1}{(x-5)(x-3)}$

$= \dfrac{(x-1)(x-5) + (x-3)(x-5) - (x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$

$= \dfrac{(x^2-6x+5) + (x^2-8x+15) - (x^2-3x+2)}{(x-1)(x-2)(x-3)(x-5)}$

$= \dfrac{x^2-11x+8}{(x-1)(x-2)(x-3)(x-5)}$

$= \dfrac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$

$= \dfrac{x-9}{(x-1)(x-3)(x-5)}$

6. **Find the square root of**
$64x^4 - 16x^3 + 17x^2 - 2x + 1$ *(Example - 3.21)*

Soln:

	8	-1	1		
8	64	-16	17	-2	1
	64				
16 -1		-16	17	-2	1
		-16	1		
16 -2 1			16	-2	1
			16	-2	1
				0	

Therefore square root of $P(x)$ is
$|18x^2 - x - 11|$

7. **Find the square root of**
$x^2 - 28x^3 + 4x^4 + 42x + 9$ *(Ex.3.8 – 1(ii))*

Soln: $P(x) = 4x^4 - 28x^3 + 37x^2 + 42x + 9$

	2	-7	-3		
2	4	-28	37	42	1
	4				
4 -7		-28	37	42	9
		-28	49		
4 -14 -3			-12	42	9
			-12	42	9
				0	

Therefore square root of $P(x)$ is
$|2x^2 - 7x - 3|$

FOR PRACTICE
(i) $x^4 - 12x^3 + 42x^2 - 36x + 9$
(ii) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

8. **If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.**
(Example - 3.22)
Soln: $P(x) = 9x^4 + 12x^3 + 28x^2 + ax +$

	3	2	4		
3	9	12	28	a	b
	9				
6 2		12	28	a	b
		12	4		
6 4 4			24	a	b
			24	16	6
				0	

Therefore square root of $P(x)$ is $3x^2 - 2x - 4$
$a - 16 = 0 \Rightarrow a = 16$
$b - 16 = 0 \Rightarrow b = 16$

9. **Find the values of a and b if the following polynomials are perfect squares**
$ax^4 + bx^3 + 361x^2 + 220x + 100$
(Ex.3.8 – 2(ii))

Soln.: $P(x) = 100 + 220x + 361x^2 + bx^3 + ax^4$

	10	11	12		
10	100	220	361	b	a
	100				
20 11		220	361	b	a
		220	121		
20 22 12			240	b	a
			240	264	144
				0	

Therefore square root of $P(x)$ is
$|10 + 11x + 12x^2|$

$b - 264 = 0 \Rightarrow b = 264$
$a - 144 = 0 \Rightarrow a = 144$

FOR PRACTICE
Find the values of a and b if the following polynomials is perfect squares
$4x^4 - 12x^3 + 37x^2 + bx + a$

10. **Find the values of m and n if the following expressions are perfect squares**
$x^4 - 8x^3 + mx^2 + nx + 16$ *(Ex.3.8 – 3(ii))*

Soln.:

$$P(x) = x^4 - 8x^3 + mx^2 + nx + 16$$

	1	-4	4		
1	1	-8	m	n	16
	1				
2 -4		-8	m	n	16
		-8	16		
2 -8 4			m-16	n	16
			8	32	16
				0	

Therefore square root of $P(x)$ is
$|x^2 - 4x + 4|$

m – 16 – 8 = 0
m – 24 = 0
m = 24

n + 32 = 0
n = -32

11. If $A=\begin{pmatrix}4&3&1\\2&3&-8\\1&0&-4\end{pmatrix}$ $B=\begin{pmatrix}2&3&4\\1&9&2\\-7&1&-1\end{pmatrix}$ and $C=\begin{pmatrix}8&3&4\\1&-2&3\\2&4&-1\end{pmatrix}$ then verify that $A+(B+C)=(A+B)+C$ *(Ex - 3.18 - 2)*

Soln.:

$$B+C=\begin{pmatrix}10&6&8\\2&7&5\\-5&5&-2\end{pmatrix}$$

$$A+(B+C)=\begin{pmatrix}4&3&1\\2&3&-8\\1&0&-4\end{pmatrix}+\begin{pmatrix}10&6&8\\2&7&5\\-5&5&-2\end{pmatrix}$$

$$=\begin{pmatrix}14&9&9\\4&10&-3\\-4&5&-6\end{pmatrix} \quad \text{............(1)}$$

$$A+B=\begin{pmatrix}6&6&5\\3&12&-6\\-6&1&-5\end{pmatrix}$$

$$\therefore (A+B)+C=\begin{pmatrix}6&6&5\\3&12&-6\\-6&1&-5\end{pmatrix}+\begin{pmatrix}8&3&4\\1&-2&3\\2&4&-1\end{pmatrix}$$

$$=\begin{pmatrix}14&9&9\\4&10&-3\\-4&5&-6\end{pmatrix} \quad \text{............(2)}$$

$\therefore$ From (1) & (2) LHS = RHS

12. Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}$ $B=\begin{pmatrix}4&0\\1&5\end{pmatrix}$ Show that $(A-B)^T=A^T-B^T$
(Ex - 3.19 – 7(iii))

Soln.:

$$A-B=\begin{pmatrix}1&2\\1&3\end{pmatrix}-\begin{pmatrix}4&0\\1&5\end{pmatrix}$$

$$=\begin{pmatrix}1-4&2-0\\1-1&3-5\end{pmatrix}$$

$$A-B=\begin{pmatrix}-3&2\\0&-2\end{pmatrix};$$

$$(A-B)^T=\begin{pmatrix}-3&8\\2&-2\end{pmatrix} \ldots(1)$$

$$A=\begin{pmatrix}1&2\\1&3\end{pmatrix};\quad A^T=\begin{pmatrix}1&1\\2&3\end{pmatrix};$$

$$B=\begin{pmatrix}4&0\\1&5\end{pmatrix};\quad B^T=\begin{pmatrix}4&1\\0&5\end{pmatrix}$$

$$A^T-B^T=\begin{pmatrix}1&1\\2&3\end{pmatrix}-\begin{pmatrix}4&1\\0&5\end{pmatrix}=\begin{pmatrix}1-4&1-1\\2-0&3-5\end{pmatrix}$$

$$A^T-B^T=\begin{pmatrix}-3&0\\2&-2\end{pmatrix}$$

Hence, verified

13. Given that $A=\begin{pmatrix}1&3\\5&-1\end{pmatrix}$ $B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}$ $C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$ verify that $A(B+C)=AB+AC$
(Ex - 3.19 - 5)

Soln.:

LHS : A (B + C)

$$=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}$$

$$=\begin{pmatrix}1\times2+3\times-1 & 1\times2+3\times6 & 1\times4+3\times5\\5\times2+(-1)\times(-1) & 5\times2-1\times6 & 5\times4-1\times5\end{pmatrix}$$

$$=\begin{pmatrix}2-3&2+18&4+15\\10+1&10-6&20-5\end{pmatrix}$$

$$=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix} \quad \text{..........(1)}$$

RHS : AB + AC

$$=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}+\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$$

$$=\begin{pmatrix}1+9&-1+15&2+6\\5-3&-5-5&10-2\end{pmatrix}+\begin{pmatrix}1-12&3+3&2+9\\5+4&15-1&10-3\end{pmatrix}$$

$$=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}+\begin{pmatrix}-11&6&11\\9&14&7\end{pmatrix}$$

$$=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix} \quad \text{............(2)}$$

$\therefore$ From (1) & (2) LHS = RHS

FOR PRACTICE

If $A=\begin{pmatrix}1&1\\-1&3\end{pmatrix}$ $B=\begin{pmatrix}1&2\\-4&2\end{pmatrix}$ $C=\begin{pmatrix}-7&6\\3&2\end{pmatrix}$

verify that $A(B+C)=AB+AC$

14. **If** $A = \left(\quad \right)$ **show that** $^{2} - 5A + 7I_2 = 0$

(Ex - 3.19 - 13)

Soln.:

$(\quad)(\quad)$

$(\quad)$

$(\quad) \quad (\quad)$

$5(\quad) \quad (\quad)$

$7(\quad) \quad (\quad)$

$(\quad) + (\quad)$

$(\quad)$

$(\quad)$

$(\quad)$

$(\quad)$

Hence, verified

15. **If** $A = \left(\quad \right) \quad = \begin{pmatrix} 1 & 2 \end{pmatrix}$

verify that $(AB)^{T} = B^{T} A^{T}$ *(Ex - 3.19 - 12)*

Soln.:

$(\quad)\begin{pmatrix} 1 & \end{pmatrix}$

$(\quad)$

$(\quad) = (\quad)$

$(\quad) \quad (\quad)$

$\begin{pmatrix} 1 & \end{pmatrix} \quad (\quad)$

$(\quad) \quad \begin{pmatrix} 2 & 2 \end{pmatrix}$

$(\quad)\begin{pmatrix} 2 & 2 \end{pmatrix}$

$(\quad)$

$(\quad)$

$(\quad)$

$(\quad)$

Hence, verified

<u>FOR PRACTICE</u>

If $A = \left(\quad \right)$ and $B = \begin{pmatrix} - & 4 \end{pmatrix}$

show that $(AB)^{T} = B^{T}A^{T}$

4. GEOMETRY
TWO MARKS QUESTIONS

1. In Fig. QA, and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ. (Example : 4.6)

Solution : ΔAOQ and ΔBOP, $\angle OAQ = \angle OBP = 90^o$

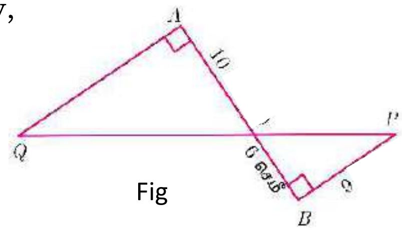
$\angle AOQ = \angle BOP$ (Vertically opposite angles)

Therefore, by AA Criterion of similarity,

$\Delta AOQ \sim \Delta BOP$

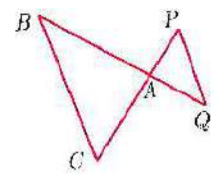
$\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$

$\frac{10}{6} = \frac{AQ}{9} \Rightarrow AQ = \frac{10 X 9}{6} = 15$ cm.



Fig

2. For Practice:
In the adjacent figure, $\Delta ACB \sim \Delta APQ$. IF BC = 8 cm, PQ = 4 cm, BA = 6.5 cm, and AP = 2.8 cm, find CA and AQ. (Exercise : 4.1-6)



3. The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ = 10 cm, find AB. (Example: 4.7)

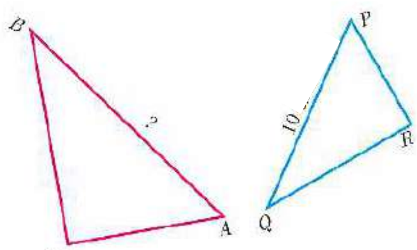
Solution : The ratio of the corresponding sides of similar triangles is same as the ratio of their perimeters.

Since $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{36}{24}$

$\frac{AB}{PQ} = \frac{36}{24} \Rightarrow \frac{AB}{10} = \frac{36}{24}$

$AB = \frac{36 X 10}{24} = 15$ cm



Fig

4. If ΔABC **is similar to ΔDEF such that BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm² . Find the area of ΔDEF. (Example : 4.8)**
Solution : Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

$$\frac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \frac{BC^2}{EF^2} \Rightarrow \frac{54}{\text{Area } \Delta DEF} = \frac{3^2}{4^2}$$

Area ΔDEF= $\frac{16X\ 54}{9}$ = 96 cm²

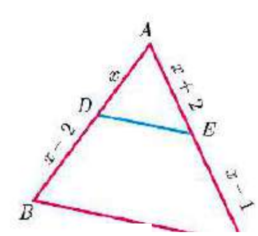
5. For Practice:
If $\Delta ABC \sim \Delta DEF$ such that are of ΔABC is 9 cm², and the area of ΔDEF is 16 cm² and BC=2.1 cm Find the length of EF. (Exercise : 4.1-8)

6. In ΔABC, if $DE \parallel BC, AD = x, DB = x-2, AE = x+2$ and $EC = x-1$ then find the lengths of the sides AB and AC. (Example 4.1-12)
Solution:

In ΔABC we have $DE \parallel BC$

By Thales theorem, we have , $\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{x}{x-2} = \frac{x+2}{x-1} \Rightarrow x(x-1) = (x-2)(x+2)$



Hence $x^2 - x = x^2 - 4 \Longrightarrow x = 4$
When $x = 4, AD = 4, DB = x - 2 = 2, AE = x + 2 = 6, EC = x - 1 = 3$
Hence , $AB = AD + DC = 4 + 2 = 6, AC = AE + EC = 6 + 3 = 9$
Therefore , $AB = 6, AC = 9$

7. For Practice:
In ΔABC D and E are points on the sides AB and AC respectively such that $DE \parallel BC$ (i) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15$ cm find AE
(ii) If $AD = 8x - 7, DB = 5x - 3, AE = 4x - 3$ and $EC = 3x - 1$ find the value of x (Exercise 4.2-1)

8. D and E are respectively the points on the sides AB and AC of a ΔABC such that $AB = 5.6$cm, $AD = 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm show that $DE \parallel BC$. (Example 4.1-13)

Solution:

$AB = 5.6$ cm $AD = 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm

$BD = AB - AD = 5.6 - 1.4 = 4.2$ cm

and $EC = AC - AE = 7.2 - 1.8 = 5.4$ cm

$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ and $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$

$\frac{AD}{DB} = \frac{AE}{EC}$

Therefore, by converse of Basic Proportionality Theorem, we have DE is parallel to BC. Hence proved.

9. In fig AD is the bisector of $\angle A$. If $BD = 4$ cm, $Dc = 3$ cm, and $AB = 6$ cm, find AC. (Example : 4.15)

Solution:

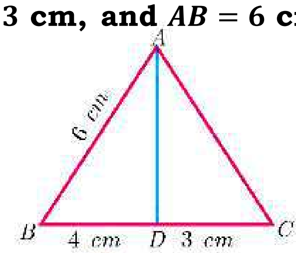


Fig.

In ΔABC, AD is the bisector of $\angle A$
By Angle Bisector Theorem
$\frac{AD}{DC} = \frac{AB}{AC}$
$\frac{4}{3} = \frac{6}{AC}$ gives $4AC = 18$, Hence, $AC = \frac{9}{2} = 4.5$ cm

10. In fig AD is the bisector of $\angle BAC$ if $AB = 10$ cm, $AC = 14$ cm, and $BC = 6$ cm, Find BD and DC. (Example. 4.16)

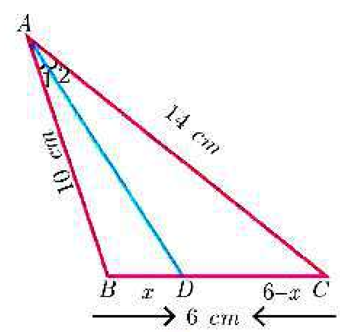


Fig.

Solution: Let $BD = x$ cm. then $DC = (6 - x)$cm

AD is the bisector of $\angle A$.

By Angle Bisector Theorem

$\frac{AB}{AC} = \frac{BD}{DC}$

$\frac{10}{14} = \frac{x}{6-x} \Longrightarrow \frac{5}{7} = \frac{x}{6-x}$

$12\,x = 30$ we get, $x = \frac{30}{12} = 2.5$ cm

Therefore $BD = 2.5$ cm, $DC = 6 - x = 6 - 2.5 = 3.5$ cm

11. Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm. (Example: 4.24)

Solution : Given OP = 5 cm, radius r = 3 cm

To find the length of tangent PT

In right angled ΔOTP

$OP^2 = OT^2 + PT^2$ (by Pythagoras theorem)

$5^2 = 3^2 + PT^2$ gives $PT^2 = 25 - 9 = 16$

Length of the tangent PT = 4 cm.

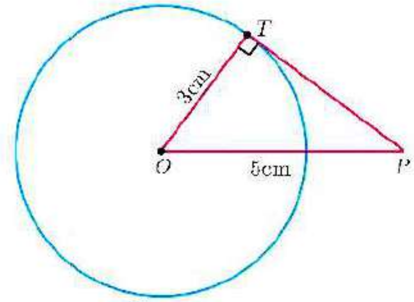


Fig.

12. For Practice:

The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle? (Exercise 4. 4 – 1)

13. In fig, O is the centre of a circle. PQ is a chord and the tangent. PR at P makes an angle of 50° with PQ. Find $\angle POQ$. (Example: 4.26)

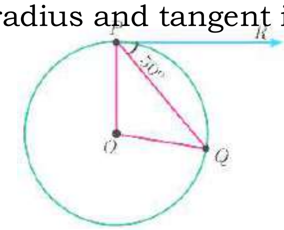
Solution: $\angle OPQ = 90^O - 50^O = 40^O$ (angle between the radius and tangent is 90°)

$OP = OQ$ (Radii of a circle are equal)

$\angle OPQ = \angle OQP = 40^O$ (ΔOPQ is isosceles)

$\angle POQ = 180^o - \angle OPQ - \angle OQP$

$\angle POQ = 180^o - 40^o - 40^o = 100^o$



Fig

14. If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle. (Example : 4.28)

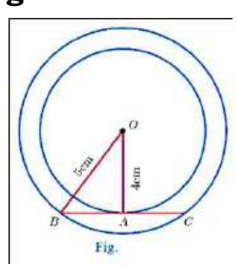
Solution : OA = 4 cm, OB = 5 cm, also $OA \perp BC$

$OB^2 = OA^2 + AB^2$

$5^2 = 4^2 + AB^2$ gives $AB^2 = 9$

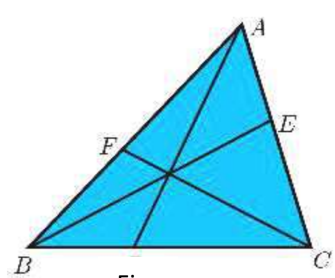
Therefore AB = 3 cm

BC = 2AB hence BC = 2 x 3 = 6 cm



15. Ceva's Theorem

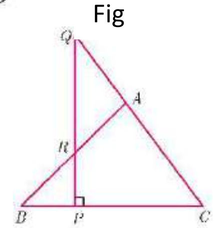
Let ABC be a triangle and let D, E, F be points on lines BC, CA, AB respectively. Then the cevians - AD, BE, CF are concurrent if and only if , $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1.$



Fig

16. Menelaus Theorem

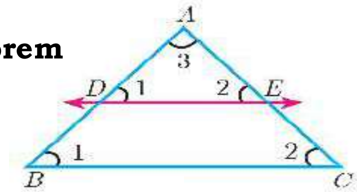
A necessary and sufficient conditions for points, P, Q, R on the respective sides BC. CA. AB (or their extension) of a triangle ABC to be collinear is that $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = -1.$



Fig

FIVE MARK QUESTIONS

1. Basic Proportionality Theorem (BPT) or Thaies Theorem



Fig

Statement

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof

Given : In ΔABC, D is a point on AB and E is a point on AC.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction :Draw a line $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle.
4.	$\Delta ABC \sim \Delta ADE$ $\frac{AB}{AD} = \frac{AC}{AE}$ $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$ $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ $\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{BD} = \frac{AE}{EC}$	By AAA Similarity Corresponding sides are proportional Split AB and AC using the points D and E On Simplification Cancelling 1 on both sides Taking reciprocal
Hence Proved		

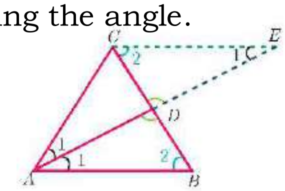
2. Angle Bisector Theorem

Statement :

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Proof :

Given : In ΔABC, AD is the internal bisector



Fig

To Prove : $\frac{AB}{AC} = \frac{BD}{CD}$

Construction :Draw a line through C parallel to AB Extend AD to meet line through C at E.

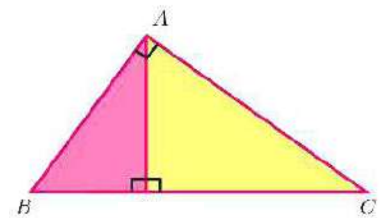
No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal
2.	ΔACE is isosceles. AC = EC . . . (1)	In ΔACE, $\angle CAE = \angle CEA$

3.	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = EC
Hence Proved.		

3. Pythagoras Theorem

Statement :

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Fig

Proof:

Given: In ΔABC, $\angle A = 90^o$

To Prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is common $\angle BAC = \angle BDA = 90^o$ Therefore $\Delta ABC \sim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ . . . (1)	Given $\angle BAC = 90^o$ and by construction $\angle BDA = 90^o$ By AA Similarity
2.	CompareΔABC and ΔDAC $\angle C$ is common $\angle BAC = \angle ADC = 90^o$ Therefore, $\Delta ABC \sim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ . . . (2)	Given $\angle BAC = 90^o$ and by construction $\angle ADC = 90^o$ By AA Similarity

Adding (1) and (2) we get

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$

$$= BC(BD + DC) = BC \times BC$$

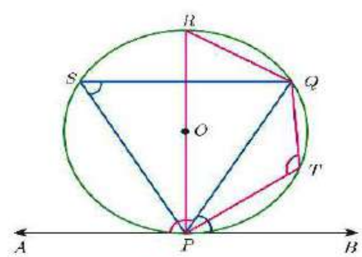
$$AB^2 + AC^2 = BC^2$$

Hence the theorem is proved.

4. Alternate Segment Theorem

Statement

If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.



Fig

Proof

Given : A circle with centre at O, tangent AB touches the circle at P and PQ is a chord. S and T are two points on the circle in the opposite sides of chord PQ.

To Prove : (i) $\angle QPB = \angle PSQ$ and (ii) $\angle QPA = \angle PTQ$
Construction : Draw the diameter POR, Draw QS and PS.

No.	Statement	Reason
1.	$\angle RPB = 90^o$ $\angle RPQ + \angle QPB = 90^o$ . . . (1)	Diameter PR is perpendicular to tangent AB
2.	In ΔRPQ $\angle PQR = 90^o$ . . . (2)	Angle is a semicircle is 90^o
3.	$\angle QRP + \angle RPQ = 90^o$ . . . (3)	In a right angled triangle, sum of the two acute angles is 90^o
4.	$\angle RPQ + \angle QPB = \angle QRP + \angle RPQ$ $\angle QPB = \angle QRP$ . . . (4)	From (1) and (3)
5.	$\angle QRP = \angle PSQ$ . . . (5)	Angles in the same segment are equal
6.	$\angle QPB = \angle PSQ$ . . . (6)	From (4) and (5); Hence (i) is proved
7.	$\angle QPB + \angle QPA = 180^0$ . . . (7)	Linear pair of angles.
8.	$\angle PSQ + \angle PTQ = 180^0$ . . . (8)	Sum of opposite angles of a cyclic quadrilateral is 180^0
9.	$\angle QPB + \angle QPA = \angle PSQ + \angle PTQ$	From (7) and (8)
10.	$\angle QPB + \angle QPA = \angle QPB + \angle PTQ$	$\angle QPB = \angle PSQ$from (6)
11.	$\angle QPA + \angle PTQ$	Hence (ii) is proved. This completes the proof.

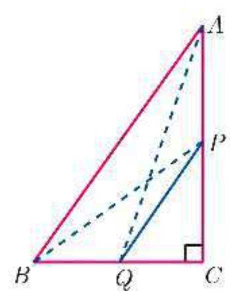
5. P and Q are the mid-point of the sides CA and CB respectively of a ΔABC right angled at C. Proved that $4(AQ^2 + BP^2) = 5AB^2$ (Ex : 4.21)

Solution:

ΔAQC is a right triangle at C, $AQ^2 = AC^2 + QC^2$ ... (1)
ΔBPC is a right triangle at C, $BP^2 = BC^2 + CP^2$ ... (2)
ΔABC is a right triangle at C, $AB^2 = AC^2 + BC^2$ ... (3)
From (1) and (2) $AQ^2 + BP^2 = AC^2 + QC^2 + BC^2 + CP^2$
$4(AQ^2 + BP^2) = 4AC^2 + 4QC^2 + 4BC^2 + 4CP^2$

$= 4AC^2 + (2QC)^2 + 4BC^2 + (2CP)^2$

$= 4AC^2 + BC^2 + 4BC^2 + AC^2$
(Since P and Q are mid points)
$= 5(AC^2 + BC^2)$ (From equation 3)
$4AQ^2 + BP^2 = 5AB^2$



Fig

6.

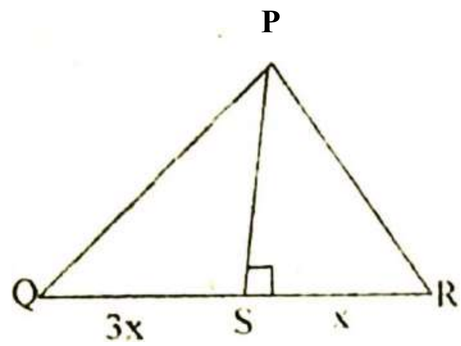
The perpendicular PS on the base QR of a ΔPQR intersects QR at S, such that QS = 3SR, prove that 2PQ² = 2PR² + QR². (Excises : 4.3-7)

Solution:

Given : $QS = 3SR$ . . . . (1)

From the figure,

$2PQ^2 = 2PR^2 + QR^2$



$SR = x$

$QS = 3SR = 3x$

$QR = QS + SR = 3x + x = 4x$

$QR = 4x \dots (A)$

$\therefore [SR = x\ QS = 3x]$

In ΔPSQ

$PQ^2 = PS^2 + QS^2$

$= PS^2 + (3x)^2$

$\Rightarrow PQ^2 = PS^2 + 9x^2 \dots (1)$

In ΔPSR

$\Rightarrow PQ^2 = PS^2 + SR^2$

$= PS^2 + x^2$

$\Rightarrow PR^2 = PS^2 + x^2 \dots (2)$

$2PR^2 = QR^2 = 2(PS^2 + x^2) + (4x)^2$ Using (1) and (2)

$= 2PS^2 + 2x^2 + 16x^2$

$= 2PS^2 + 18x^2$

$= 2(PS^2 + 9x^2)$

$= 2PQ^2$ (From (1)]

$2^2 + {}^2 = 2^2$ Hence Proved.

7. **In figure, ABC is right angled triangle with right angle at B and points D,E trisect BC, Prove that $8AE^2 = 3AC^2 + 5AD^2$ (Excises : 4.3-8)**

Solution:

$8AE^2 = 3AC^2 + 5AD^2$

From the figure,

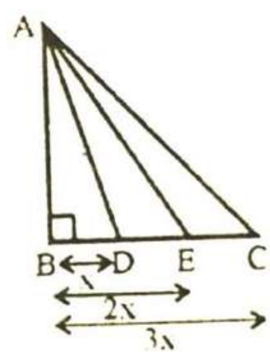
Assume that

BD = DE = EC

Now $BD = x$

$BE = 2x$

$BC = 3x$



In ΔABD By Pythagoras theorem,

$AD^2 = AB^2 + BD^2$

$\Rightarrow AD^2 = AB^2 + x^2 \dots (1)$ $[\because BD = x]$

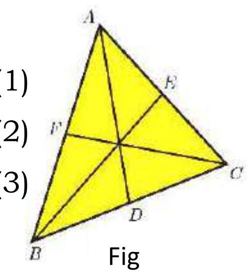
In ΔABE

$AE^2 = AB^2 + BE^2$

$= AB^2 + (2x)^2 = AB^2 + 4x^2$ $[\because BE = 2x]$
$AE^2 = AB^2 + 4x^2 \ldots (2)$
In ΔABC
$AC^2 = AB^2 + BC^2$
$= AB^2 + (3x)^2$ $[\because BC = 3x]$
$AC^2 = AB^2 + 9x^2 \ldots (3)$
$3AC^2 + 5AD^2$
$= 3(AB^2 + 9x^2) + 5(AB^2 + x^2)$ Using (2)
$= 3AB^2 + 27x^2 + 5AB^2 + 5x^2$
$= 8AB^2 + 32x^2$
$= 8(AB^2 + 4x^2)$
$= 8AEB^2$
$\therefore 3AC^2 + 5AD^2 = 8AE^2$ Hence Proved.

8. Show that in a triangle, the medians are concurrent (Example : 4.32)
Solution:

Medians are line segments joining each vertex to the medpoint of the corresponding opposite sides.
Thus medians are the cevians where D, E, F are midpoints of BC, CA and AB respectively



Fig

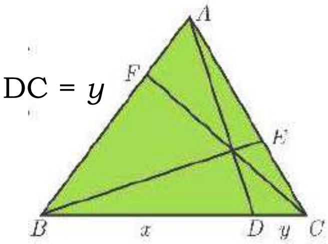
Since D is a midpoint of BC, BD = DC so $\frac{BD}{DC} = 1$ . . . (1)
Since E is a midpoint of CA, CE = EA so $\frac{CE}{EA} = 1$ . . . (2)
Since F is a midpoint of AB, AF = FB so $\frac{AF}{FB} = 1$ . . . (3)
Thus, multiplying (1), (2) and (3) we get,

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1 \times 1 \times 1 = 1$$

And so, Ceva's theorem is satisfied.
Hence the Medians are concurrent.

9. For Practice
Show that the angle bisectors of a triangle are concurrent. (Ex:4.4-9)

10. Suppose AB, AC and BC have lengths 13, 14, and 15 respectively. If $\frac{AF}{FB} = \frac{2}{5}$ and $\frac{EC}{EA} = \frac{5}{8}$ Find BD and DC (Example : 4.33)
Solution:



Fig

Given that AB = 13, AC = 14 and BC = 15. Let DB = x and DC = y
Using Ceva's theorem, we have , $\frac{DB}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ . . . (1)
Substitute the values of $\frac{AF}{FB}$ and $\frac{EC}{EA}$ in (1)
We have $\frac{DB}{DC} \times \frac{5}{8} \times \frac{2}{5} = 1$
$\frac{x}{y} \times \frac{10}{40} = 1$ we get $\frac{x}{y} \times \frac{1}{4} = 1$ Hence $x = 4y$. . . . (2)
$BC = BD + DC = 15$, so $x + y = 15$ . . . (3)
From (2) using $x = 4y$ in (3) we get $4y + y = 15$ gives $5y = 15$ then $y = 3$
Substitute $y = 3$ in (3) we get $x = 12$ Hence, $BD = 12, DC = 3$

5.COORDINATE GEOMETRY

FORMULAS:

1. Distance between two points

   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. Midpoint of two points

   $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

3. Centroid of a triangle

   $= \left(\frac{x1+x2+x3}{3}, \frac{x1+x2+x3}{3}\right)$

4. Section Formula (Internal Division)

   $= \left(\frac{mx2+nx1}{m+n}, \frac{my2+ny1}{m+n}\right)$

5. Section Formula (External Division)

   $= \left(\frac{mx2-nx1}{m-n}, \frac{my2-ny1}{m-n}\right)$

6. Area of Triangle

   $= \frac{1}{2}\begin{vmatrix} X_1 & X_2 & X_3 & X_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$ sq.units

7. Area of Quadrilateral

   $= 1/2\begin{vmatrix} X_1 & X_2 & X_3 & X_4 & X_1 \\ y_1 & y_2 & y_3 & y_4 & y1 \end{vmatrix}$ sq.units

8. If two points given Slope

   $m = \frac{y_2 - y_1}{x_2 - x_1}$

9. If angle is given, Slope $m = \tan\vartheta$

10. If equation of a straight line is given, Slope $m = \frac{-\,coefficient\ of\ x}{coefficient\ of\ y}$

11. If two lines are parallel

    $m_1 = m_2$

12. If two lines are perpendicular

    $m_1 \times m_2 = -1$

13. Equation of a straight line parallel to x axis, Then y =b
14. Equation of a straight line parallel to y axis, Then x = a
15. Equation of a straight line which is parallel to the straight line ax + by +c = 0 is ax +by +k = 0
16. Equation of a straight line which is perpendicular to the straight line ax + by +c = 0 is bx – ay + k = 0
17. Slope – Intercept form y = mx + c
18. One Point – slope form

    $y - y1 = m(x - x1)$

19. Two point form

    $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

20. Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

TWO MARK QUESTIONS

1) Find the area of the triangle whose vertices are (1,-1), (-4,6) (-3,-5) [EX :5.1(1-i)]

solution A(1,-1), B(-4,6), C(-3,-5)

$(X_1, y_1) = (1,-1)$

$(X_2, y_2) = (-4,6)$

$(X_3, y_3) = (-3,-5)$

The area of Δ ABC is $\frac{1}{2}\begin{vmatrix} X_1 & X_2 & X_3 & X_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

$=\frac{1}{2}\begin{vmatrix} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{vmatrix}$

= ½ [(6 + 20 + 3) – (4 –18 – 5)]

= ½ [29 – (-19)]

= ½ [29 +19]

= ½ [48]

= 24 sq.units

For Practice

2) Find the area of the triangle formed by the points..

. (-3,5), (5,6), (5,-2) [**Eg: 5.1]**

(-10,-4), (-8,-1), (-3,-5) [**EX:5.1(1)]**

3) The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such trinagle has the vertices at (-3,2), (-1,-1) and (1,2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

4) Determine whether the sets of points are collinear? (-1/2,3), (-5,6) ,(-8,8)

[EX :5.1(2-i)]

Solution A(-1/2,3), B(-5,6) C(-8,8)

$(X_1, y_1) = (-1/2,3)$,

$(X_2, y_2) = (-5,6)$

$(X_3, y_3) = (-8,8)$

The area of Δ ABC is $\frac{1}{2}\begin{vmatrix} X_1 & X_2 & X_3 & X_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

$=\frac{1}{2}\begin{vmatrix} -1/2 & -5 & -8 & -1/2 \\ 3 & 6 & 8 & 3 \end{vmatrix}$

= ½ [(-3 -40 -24) – (-15 –48 – 4)]

= ½ [-67 – (-67)]

= ½ [-67 +67]

= ½ [0] = 0

Therefore, the given points are collinear.

For Practice

5) Determine whether the set of points are collinear?

(i) P(-1.5,3), Q(6,-2) R(-3,4)

(ii) (a,b+c), (b,c+d) and (c,a+d) **[EX:5.1(2)]**

6)If the area of the triangle formed by the vertices A (-1,2),B (k ,-2) and C(7 ,4)(taken in order) is 22sq.units, find the values of k. [**Eg: 5.3]**

Solution $(x_1, y_1) = (-1, 2)$

$(x_2, y_2) = (k, -2)$

$(x_3, y_3) = (7, 4)$

The area of Δ ABC = 22 sq.units

$$\frac{1}{2}\begin{vmatrix} X_1 & X_2 & X_3 & X_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = 22$$

$$\frac{1}{2}\begin{vmatrix} -1 & k & 7 & -1 \\ 2 & -2 & 4 & 2 \end{vmatrix} = 22$$

$\frac{1}{2}$ [(2 +4k +14) – (2k –14 – 4)] = 22

1/2[(4k + 16) –(2k -18)] = 22

4k +16 -2k +18 = 44

2k + 34 = 44

2k = 44 -34

2k = 10

K = 5

For Practice

7) Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of 'p' **[EX:5.1(3)]**

	Vertices	**Area(Sq.units)**
i)	(0,0), (p,8),(6,2)	20
ii)	(p,p), (5,6), (5,-2)	32

8) In each of the following, find the value of 'a' for which the given points are collinear. (2,3), (4,a), (6,-3) **[EX:5.1(4)]**

Solution A(2, 3), B(4,a), C(6,-3)

$(X_1, y_1) = (2, 3)$,

$(X_2, y_2) = (4, a)$

$(X_3, y_3) = (6, -3)$

The area of Δ ABC = 0 sq.units

$$\frac{1}{2}\begin{vmatrix} X_1 & X_2 & X_3 & X_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = 0$$

$$\frac{1}{2}\begin{vmatrix} 2 & 4 & 6 & 2 \\ 3 & a & -3 & 3 \end{vmatrix} = 0$$

$\frac{1}{2}$ [(2a -12 +18) – (12 +6a - 6)] = 0

2a + 6 – (6a + 6) = 0

2a + 6 – 6a -6 = 0

-4a = 0

a = 0

For Practice

9) In each of the following, find the value 'a' for which the given points are collinear.

(a, 2-2a), (-a +1, 2a) and (-4-a, 6-2a)

[EX:5.1(4)]

10) What is the slope of a line whose inclination is 30^0 **[Eg: 5.8]**

Solution $\theta = 30^0$

Slope m = $\tan\theta$

$$m = \tan 30° = \frac{1}{\sqrt{3}}$$

For Practice

11) What is the slope of a line whose inclination with positive direction of x-axis is

(i) 90^0 (ii) 0^0 **[EX:5.2(1-i)]**

12) What is the inclination of a line whose slope is √3 **[Eg:5.8]**

Solution

m = √3

$$m = \tan\theta$$

$$\tan\theta = \sqrt{3}$$

$\tan 60^0 = \sqrt{3}$

$$\theta = 60^\circ$$

For Practice

13) What is the inclination of a line whose slope is **[EX:5.2(2)]**

(i) 0 (ii) 1

14) Find the slope of a line joining the given points (-6,1), (-3,2) **[Eg:5.9]**

Solution

(-6, 1) , (-3,2)

The slope m $= \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{2-1}{-3+6} = \frac{1}{3}$

m = 1/3

For Practice

15) Find the slope of a line joining the points. **[Eg:5.9 , EX:5.2(3)]**

(i) (14,10) and (14,-6)
(ii) (-1/3,1/2) and (2/7, 3/7)
(iii) (5, √5) with the Origin
(iv) (sinϑ , -cosϑ), (- sinϑ , cosϑ)

16) The line r passes through the points (-2,2) and (5,8) and the line s passes through the points (-8,7) and (-2,0). Is the line r perpendicular to s **[Eg:5.10]**

Solution

(-2,2) , (5,8)

The slope of the line r,

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{8-2}{5+2} = \frac{6}{7}$$

(-8,7), (-2,0)

The slope of the line s ,

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_2 = \frac{0-7}{-2+8} = \frac{-7}{6}$$

The product of the slopes $= \frac{6}{7} \times \frac{-7}{6}$

$m_1 \times m_2 = -1$

Therefore, the line r is perpendicular to line s.

For Practice

17) The line p passes through the points (3 ,-2) , (12,4) and the line q passes through the points (6 ,-2) and (12 ,2). Is p parallel to q ? **[Eg:5.11]**

18) What is the slope of a line perpendicular to the line joining A(5 ,1) and P where P is the midpoint of the segment joining (4,2) and (-6, 4)

[EX :5 .2(4)]

19) Show that the points (-2,5) , (6,-1) and (2,2) are collinear. **[Eg:5.12]**

Solution

The vertices are A(-2,5) , B(6,-1), C(2,2)

A(-2,5) , B(6,-1)

$(X_1 , y_1) = (-2, 5)$

$(X_2 , y_2) = (6 ,-1)$

Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-5}{6+2} = \frac{-6}{8}$

$= \frac{-3}{4}$

B(6,-1), C(2,2)

Slope of BC $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2+1}{2-6} = \frac{3}{-4}$

$= \frac{-3}{4}$

Slope of AB = Slope of BC

Hence the points A ,B and C are collinear..

For Practice

20) Show that the given points are collinear : (-3,-4), (7,2) and (12,5)

[EX :5 .2(5)]

21) If the three points (3,-1), (a ,3) and (1,-3) are collinear, find the value of a. **[EX :5 .2(6)]**

22) The line through the points (-2, a) and (9 ,3) has slope $\frac{-1}{2}$. Find the value of a. **[EX :5 .2(7)]**

23) Find the equation of a straight line passing through (5,7) and is

(i) Parallel to X axis
(ii) Parallel to Y axis **[Eg:5.17]**

Solution

(i) The equationof any straight line parallel to X axis is y = b

(a , b) = (5,7)

b = 7

The required equation of the line is **y =7**

(ii) The equation of any straight line parallel to Y axis is x = a

(a , b) = (5,7)

a = 5

The required equation of the line is **x = 5**

For Practice

24) Find the equation of a straight line passing through the mid –point of a line segment joining the points (1,-5), (4 ,2) and parallel to (i) X axis (ii) Y axis

[EX :5 .3(1)]

25) The equation of a straight line is 2 (x –y) + 5 = 0. Find its slope, inclination and intercept on the Y axis. **EX :5 .3(2)]**

Solution

2 (x –y) + 5 = 0

2x – 2y + 5 = 0

Slope m = $\frac{-a}{b}$

m = $\frac{-2}{-2}$

m = 1

Y intercept C $= \frac{-5}{-2}$

$C = \frac{5}{2}$

$1 = \tan\vartheta$, $m = \tan\theta$

$\tan 45 = \tan\vartheta$

Inclination $\vartheta = 45^0$

For Practice

26) Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$. **[Eg:5.19]**

27) Find the equation of a line whose inclination is 30^0 and making an intercept -3 on the Y axis. **[EX :5 .3(3)]**

Solution

$\vartheta = 30^0$

Y - intercept C = - 3

$m = \tan\theta$, $m = \tan 30^0$

$m = 1/\sqrt{3}$

The required equation of the line

$y = mx + c$

$y = \frac{1}{\sqrt{3}}x - 3$

$\sqrt{3}y = x - 3\sqrt{3}$

$x - \sqrt{3}y - 3\sqrt{3} = 0$

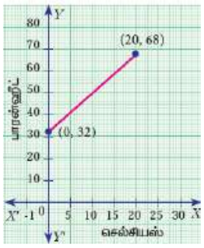
For Practice

28) Find the equation of a straight line whose **[Eg:5.18]**

(i) Slope = 5, y intercept c = -9

(ii) Inclination is 45^0 and y intercept is 11.

29) The graph relates temperatures y (in Fahrenheit degree) to temperatures x (in Celsius degree) (a) Find the slope and y intercept (b) Write an equation of the line (c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is 25^0 Celsius? **[Eg:5.20]**



30) Find the equation of a line passing through the point (3,-4) and having slope $\frac{-5}{7}$ **[Eg:5.21]**

Solution

$(x_1, y_1) = (3,-4)$

$m = \frac{-5}{7}$

The equation of the point-slope form of the straight line is

$$y - y_1 = m(x - x_1)$$

$Y + 4 = \frac{-5}{7}(x - 3)$

$7(y + 4) = -5(x - 3)$

$7y + 28 = -5x + 15$

$5x + 7y + 28 - 15 = 0$

$5x + 7y + 13 = 0$

For Practice

31) Find the equation of a straight line which has slope -5/4 and passing through the point (-1,2). **[EX :5 .3(10)]**

32) The hill in the form of a right triangle has its foot at (19 ,3). The inclination of the hill to the ground is

45^0. Find the equation of the hill joining the foot and top. [**EX :5 .3(6)**]

33) Find the equation of a straight line passing through (5 ,-3) and (7, -4). என்ற **[Eg:5.23]**

Solution

(5 ,-3) , (7, -4)

The equation of a straight line passing through the two points (x_1 , y_1), (x_2,y_2)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y+3}{-4+3} = \frac{x-5}{7-5}$$

$$\frac{y+3}{-4+3} = \frac{x-5}{7-5}$$

$$\frac{y+3}{-1} = \frac{x-5}{2}$$

2(y + 3) = -1(x – 5)

2y + 6 = -x + 5

X + 2y + 1 = 0

For Practice

34) Find the equation of aline through the given pair of points [**EX :5 .3(7)**]

(i) (2 ,2/3) and (-1/2, -2)

(ii) (2 ,3) and (-7, -1)

35) A cat is located at the point (-6,-4) in xy plane. A bottle of milk is kept at (5,11). The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk. [**EX :5 .3(8)**]

36) Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from (6 ,10) to (14 ,12), find the equation of the rod joining the buildings?. **[Eg:5.24]**

37) Find the intercepts made by the line 4x –9y + 36 = 0 on the coordinate axes. **[Eg:5.26]**

Solution

4x –9y + 36 = 0

4x –9y = - 36

Dividing by - 36 ,

$$\frac{x}{-9} - \frac{y}{-4} = 1$$

$$\frac{x}{-9} + \frac{y}{4} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1,$$

x intercept a = -9 , y intercept b = 4

For Practice

38) Find the intercepts made by the following lines on the coordinate axes. .

(i) 3x -2y -6 = 0

(ii) 4x + 3y + 12 = 0 [**EX :5 .3(13)**]

39) Find the equation of a line whose intercepts on the x and y axes are given below. [**EX :5 .3(12)**]

4 ,-6

Solution

x intercept a = 4 , y intercept b = -6

The required equation of a line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{4} + \frac{y}{-6} = 1$$

$$\frac{-6x+4y}{-24} = 1$$

-6x + 4y = -24

Dividing by -2 , we get 3x - 2y = 12

3x - 2y – 12 = 0

For Practice

40)) Find the equation of a line whose intercepts on the x and y axes are given below. [**EX :5 .3(12)]**

-5 ,3/4

41) Find the slope of the following straight lines 5y – 3 = 0. [**EX :5 .4(1)]**

Solution

$5y - 3 = 0$

$m = \frac{-\,coefficient\ of\ x}{coefficient\ of\ y}$

$m = \frac{0}{5}$

$m = 0$

For Practice

42) Find the slope of the following straight lines.

(i) 7x -3/17 = 0 [**EX :5 .4(1)]**

(ii) 6x + 8y + 7 = 0 **[Eg:5.30]**

43) Find the slope of the line which is (i)parallel to 3x - 7y = 11 (ii) perpendicular to 2x - 3y + 8 = 0. **[Eg:5.31]**

Solution

$3x - 7y - 11 = 0$

$m = \frac{-\,coefficient\ of\ x}{coefficient\ of\ y}$

$m = \frac{-3}{-7}$

$m = \frac{3}{7}$

Slope of any line parallel to 3x-7y =11 is $\frac{3}{7}$.

$2x - 3y + 8 = 0$

Slope $m = \frac{-2}{-3}$

$m = \frac{2}{3}$

Slope of any line perpendicular to 2x - 3y+8 =0 is $\frac{-3}{2}$

For Practice

44) Find the slope of the line which is (i)parallel to y = 0.7x - 11 (ii) perpendicular to the line x = -11

[**EX :5 .4(2)]**

45) Check whether the given lines are parallel $\frac{x}{3} + \frac{y}{4} + \frac{1}{7} = 0$ மற்றும் $\frac{2x}{3} + \frac{y}{2} + \frac{1}{10} = 0$. [**EX :5 .4(3)]**

Solution

$\frac{x}{3} + \frac{y}{4} + \frac{1}{7} = 0$

$m = \frac{-\,coefficient\ of\ x}{coefficient\ of\ y}$

$m_1 = \frac{-1/3}{1/4}$

$= \frac{-1}{3} \times \frac{4}{1}$

$m_1 = -4/3$

$\frac{2x}{3} + \frac{y}{2} + \frac{1}{10} = 0$

$m_2 = \frac{-2/3}{1/2}$

$= \frac{-2}{3} \times \frac{2}{1}$

$m_2 = -4/3$

$m_1 = m_2 = -4/3$

Hence the two lines are parallel..

For Practice

46) Check whether the given lines are parallel $2x + 3y - 8 = 0$, $4x + 6y + 18 = 0$ **[Eg:5.32]**

47) Check whether the given lines are perpendicular

$x - 2y + 3 = 0$, $6x + 3y + 8 = 0$

[Eg:5.33]

Solution

$x - 2y + 3 = 0$

$$m = \frac{-\,coefficient\ of\ x}{coefficient\ of\ y}$$

$$m_1 = \frac{-1}{-2} = \frac{1}{2}$$

$6x + 3y + 8 = 0$

$$m_2 = \frac{-6}{3} = -2$$

$$m_1 \times m_2 = \frac{1}{2} \times -2$$

$m_1 \times m_2 = -1$

Hence, the two straight lines are perpendicular.

For Practice

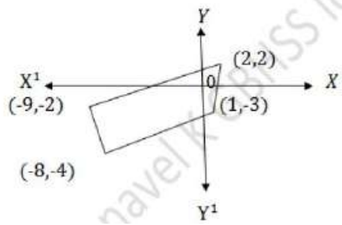
48) Check whether the given lines are perpendicular $5x + 23y + 14 = 0$, $23x - 5y + 9 = 0$ [**EX :5 .4(3)]**

49) If the straight lines $12y = -(p + 3)x + 12$, $12x - 7y = 16$ are perpendicular then find 'p' . [**EX :5 .4(4)]**

FIVE MARKS QUESTIONS

1) Find the area of the quadrilateral whose vertices are at (-9 ,-2), (-8, -4), (2,2) and (1, -3). [**EX :5 .1(5)]**

Solution



A(-9 ,-2), B(-8, -4), C(1, -3), D (2,2)

$(X_1 , y_1) = (-9, -2)$

$(X_2 , y_2) = (-8 , -4)$

$(X_3 , y_3) = (1, -3)$

$(X_4 , y_4) = (2 , 2)$

Area of quadrilateral ABCD

$$= 1/2 \begin{vmatrix} X_1 & X_2 & X_3 & X_4 & X_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} \text{ sq.units}$$

$$= 1/2 \begin{vmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{vmatrix}$$

$= 1/2 [(36+24+2-4) – (16-4-6-18)]$

$= ½ [58 – (-12)]$

$= ½ (58 +12)$

$= 1/2 \times 70$

$= 35$

Area of quadrilateral ABCD = 35 sq.units

For Practice

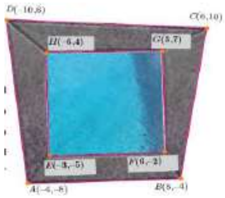
2)Find the area of the quadrilateral whose vertices are at **[EX :5 .1(5), Eg:5.6]**

1.(i)(-9 ,0), (-8, 6), (-1,-2) and (-6, -3)

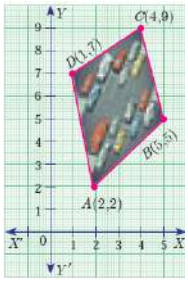
(ii) (8 ,6), (5, 11), (-5,12) and (-4, 3)

3) Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are taken in the order (-4 ,-2), (-3, k), (3,-2) (2, 3). **[EX :5 .1(6)]**

4) In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.. **[EX :5 .1(9)]**



5) The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost 1300 per square feet. What will be the total cost for making the parking lot? **[Eg:5.7]**



6) A triangular shaped glass with vertices at A(-5 ,-4), B(1, 6) and C(7,-4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied. **[EX :5 .1(10)]**

7) If the points A(-3 ,9), B(a, b) and C(4,-5) are collinear and if a + b = 1, then find a and b.**[EX :5 .1(7)]**

Solution

A(-3 ,9), B(a, b) and C(4,-5)

If the points are collinear, Area of triangle ABC = 0

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{vmatrix} = 2 \times 0$$

[(-3b -5a + 36) – (9a + 4b + 15)] = 0

-3b -5a + 36 – 9a - 4b - 15= 0

-14a -7b +21 = 0

Dividing by -7

2a + b - 3 = 0

2a + b = 3 , a + b =1 (Given)

2a + b = 3 ------(1)

a + b = 1-------(2)

(-) (-) (-)

a = 2

Sub a =2 in (1) , we get 2 X 2 + b = 3

4 + b = 3 , b = -1

a = 2 , b = -1

For Practice

8) If the points P(-1, -4), Q(b, c) and R(5,-1) are collinear and if 2b + c = 4, then find the values of b and c . **[Eg:5.4]**

9) The line through the points (-2 ,6) and (4, 8) is perpendicular to the line through the points (8 ,12) and (x ,24).

[EX :5 .2(8)]

Solution

A(-2 ,6), B(4, 8), C(8 ,12), D(x ,24)

A(-2 ,6), B(4, 8)

Slope of AB $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{8-6}{4+2}$

$= \frac{2}{6} = \frac{1}{3}$

$m_1 = \frac{1}{3}$

C (8 ,12) , D (x ,24)

Slope of CD $m_2 = \frac{24-12}{x-8}$

$= \frac{12}{x-8}$

If the lines are perpendicular,

$m_1 \times m_2 = -1$

$\frac{1}{3} \times \frac{12}{x-8} = -1$

$\frac{4}{x-8} = -1$

$4 = -1(x-8)$

$4 = -x + 8$

$x = 8 - 4$

$x = 4$

10) show that the given points (1 ,-4), (2, -3) and (4,-7) form a right angled triangle and check whether they satisfies Pythagoras theorem. [**EX :5 .2(9)]**

Solution A(1 ,-4), B(2, -3) and C(4,-7)

Slope of the line m $= \frac{y_2 - y_1}{x_2 - x_1}$

The slope of AB = $\frac{-3-(-4)}{2-1} = \frac{-3+4}{1} = 1$

The slope of BC = $\frac{-7-(-3)}{4-2} = \frac{-7+3}{2} = \frac{-4}{2}$ $= -2$

The slope of CA = $\frac{-4-(-7)}{1-4} = \frac{-4+7}{-3} = \frac{3}{-3} = -1$

Slope of AB x Slope of CA = 1 x -1 = -1

AB is perpendicular to CA angle A= 90^0

BC - Hypotenus

Δ ABC is Right angled triangle

The distance between the two points $(x_1, y_1), (x_2, y_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

A(1 ,-4), B(2, -3)

AB $= \sqrt{(2-1)^2 + (-3-(-4))^2}$

$= \sqrt{1^2 + (-3+4)^2}$

$\sqrt{1+1} = \sqrt{2}$

B(2, -3) C(4,-7)

BC $= \sqrt{(4-2)^2 + (-7-(-3))^2}$

$= \sqrt{(2)^2 + (-7+3)^2}$

$= \sqrt{4 + (-4)^2}$

$= \sqrt{4+16} = \sqrt{20}$

C(4,-7), A(1 ,-4)

CA $= \sqrt{(1-4)^2 + (-4-(-7))^2}$

$= \sqrt{(-3)^2 + (-4+7)^2}$

$= \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9} = \sqrt{18}$

By Pythagoras theorem,

$BC^2 = AB^2 + AC^2$

$(\sqrt{20})^2 = (\sqrt{2})^2 + (\sqrt{18})^2$

$20 = 2 + 18$

They satisfied Pythagoras theorem.

For Practice

11) show that the given points . L(0 ,5), M(9,12) and N(3,14) form a right angled triangle and check whether they satisfies Pythagoras theorem [**EX :5 .2(9)]**

12) Without using Pythagoras theorem, show that the points (1 ,-4), (2,-3) and (4,-7) form a right angled triangle. **[Eg:5.15]**

13) If the points A(2,2), B(-2,-3), C(1,-3) and D(x, y) form a parallelogram then find the value x and y. [**EX :5 .2(11)]**

Solution

A(2,2), B(-2,-3), C(1,-3) and D(x,y)

Slope of AB = Slope of CD

Slope m $= \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{-3-2}{-2-2} = \frac{y+3}{x-1}$

$\frac{-5}{-4} = \frac{y+3}{x-1}$

$\frac{5}{4} = \frac{y+3}{x-1}$

5(x -1) = 4 (y +3)

5x – 5 = 4y + 12

5x – 4y = 12+5

5x – 4y =17 ------------ (1)

B(-2,-3), C(1,-3) , A(2,2), D(x,y)

Slope of BC = Slope of AD

$\frac{-3+3}{1+2} = \frac{y-2}{x-2}$

$\frac{0}{3} = \frac{y-2}{x-2}$

3(y -2) = 0

3y - 6 = 0

3y = 6

y = 2

Sub y = 2 in (1)

5x – 4 x 2 = 17

5x – 8 = 17

5x = 17 + 8

5x = 25

X = 5

X = 5 , y = 2

For Practice

14) Show that the given points form a parallelogram: A (2.5, 3.5), B(10,-4), C(2.5,-2.5) and D(-5,5). [**EX :5 .2(10)]**

15) Let A(3 , -4) , B(9 , -4), C(5, -7) and D(7,-7). Show that ABCD is a trapezium.[**EX :5 .2(12)]**

16) A quadrilateral has vertices at A(-4 ,-2) , B(5 -1), C(6, 5) and D(-7,6). Show that the mid-points of its sides form a parallelogram. [**EX :5 .2(13)]**

17) Let A(1 ,-2) , B(6 -2), C(5, 1) and D(2,1)be four points

(i) Find the slope of the line segments a) AB b) CD

(ii) Find the slope of the line segments a) BC b) AD

What can you deduce from your answer. **[Eg:5.13]**

18) Find the equation of aline passing through the point A(1,4) and perpendicular to the line joining points (2 5) and (4, 7). **[Eg:5.22]**

Solution

B(2 5), C(4, 7)

Slope of BC , m $= \frac{y_2 - y_1}{x_2 - x_1}$

m = $\frac{4-2}{7-5} = \frac{2}{2} = 1$

The required line is perpendicular to BC, m x 1 = -1, m = -1 m = -1 , A(1,4)

The equation of the required straight line is

$Y - y_1 = m(x - x_1)$

$Y - 4 = -1(x - 1)$

$Y - 4 = -x + 1$

$x + y - 5 = 0$

For Practice

19) Find the equation of a line passing through (6,-2) and perpendicular to the line joining the points (6 7) and (2, -3). [**EX :5 .4(6)]**

20) Find the equation of a straight line passing through the point P(-5,2) and parallel to the line joining the points Q(3, -2) and R(-5, 4) [**EX :5 .4(5)]**

21) A(-3,0), B(10,-2) and C(12,3) are the vertices of ΔABC. Find the equation of the altitude through A and B [**EX :5 .4(7)]**

22) Find the equation of a straight line (i) passing through (1, -4) and has intercepts which are in the ratio 2:5 [**EX :5 .3(14)]**

Solution

Ratio of intercepts a: b = 2:5

$$\frac{a}{b} = \frac{2}{5}$$

$$a = \frac{2b}{5}$$

The equation of the required line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2b/5} + \frac{y}{b} = 1$$

$$\frac{5x}{2b} + \frac{y}{b} = 1$$

$$\frac{5x+2y}{2b} = 1$$

$5x + 2y = 2b$ → (1)

The line $5x + 2y = 2b$ pass through the point (1,-4)

$5 \times 1 + 2 \times (-4) = 2b$

$5 - 8 = 2b$

$2b = -3$

$b = -3/2$

Sub $b = -3/2$ in **(1)**

$5x + 2y = 2b$

$5x + 2y = 2(-3/2)$

$5x + 2y + 3 = 0$

For Practice

23) Find the equation of aline which passes through (5, 7) and makes intercepts on the axes equal in magnitude but opposite in sign.**[Eg:5.25]**

24) A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through (-3,8). Find its equation..**[Eg:5.28]**

25)Find the equation of a straight line(ii) passing through (-8, 4) and making equal intercepts on the coordinate axes.. [**EX :5 .3(14)]**

26) Find the equation of the median and altitude of Δ ABC through A where the vertices are A(6, 2), B(-5,-1) and C(1,9) [**EX :5 .3(9)]**

Solution

B(-5,-1) ,C(1,9)

Midpoint of BC,

$$D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$D = \left(\frac{-5+1}{2}, \frac{-1+9}{2} \right)$$

$$D = \left(\frac{-4}{2}, \frac{8}{2} \right)$$

D = (-2 , 4)

The equation of the median AD

A(6,2), D(-2 , 4)

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{2} = \frac{x-6}{-8}$$

-8 (y – 2) = 2 (x – 6)

-8y + 16 = 2x -12

2x + 8y – 28 = 0

Dividing by 2

x + 4y – 14 = 0

The equation of the altitude

B(-5 ,-1), C(1, 9)

The slope of BC $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{9+1}{1+5}$$

$$= \frac{10}{6} = 5/3$$

The slope of altitude m = -3/5

m = -3/5 , A(6,2)

Equation of the altitude AD is

$y - y_1 = m (x - x_1)$

y – 2 = -3/5 (x – 6)

5(y -2) = -3 (x – 6)

5y – 10 = -3x + 18

3x + 5y – 28 = 0

Equation of Median : x + 4y -14 = 0

Equation of Altitude : 3x + 5y -28 = 0

27)Find the equation of the perpendicular bisector of the line joining the points A(-4,2) and B(6,-4)

[**EX :5 .4(8)**]

Solution

A(-4 , 2) B(6,-4)

Midpoint of AB = $D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= D\left(\frac{-4+6}{2}, \frac{2-4}{2} \right)$$

= D (2/2 , -2/2)

= D (1 , -1)

A(-4 , 2), B(6,-4)

Slope of AB $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-4-2}{6+4}$$

$$= \frac{-6}{10} = \frac{-3}{5}$$

Slope of altitude $m = \frac{5}{3}$

$m = \frac{5}{3}$, D (1, -1)

The equation of the perpendicular bisector is

$y - y_1 = m (x - x_1)$

y + 1 = 5/3 (x – 1)

3 (y + 1) = 5 (x – 1)

3y + 3 = 5x – 5

5x – 5 - 3y - 3 = 0

5x – 3y – 8 = 0

28) Find the equation of a straight line through the intersection of lines

7x + 3y = 10, 5x - 4y = 1 and parallel to the line 13x + 5y + 12 = 0 _**[EX :5 .4(9)]**

Solution

7x + 3y = 10 ⟶ (1)

5x - 4y = 1 ⟶ (2)

1 x 4 , **2** x 3 ⟶

28x + 12y = 40

15x – 12y = 3

----

43 x = 43

x = 43/43

x = 1

Sub x = 1 in **(1)**

7x + 3y = 10

7 x 1 + 3y = 10

7 + 3y = 10

3y = 10 -7

3y = 3

y = 1

The point of intersection is (1 ,1)

Equation of the line parallel to 13x + 5y + 12 = 0 is 13x + 5y + k = 0.

This line passes through (1,1)

13 x 1 + 5 x 1 + k = 0

13 + 5 + k = 0

18 + k = 0

k = -18

Sub k = -18 in 13x + 5y + k = 0

13x + 5y - 18 = 0

Therefore, The equation of the line is

13x + 5y - 18 = 0

For Practice

29) Find the equation of a straight line through the intersection of lines 5x - 6y = 2, 3x + 2y = 10 and perpendicular to the line 4x - 7y+13 = 0 **[EX :5 .4(10)]**

30) Find the equation of a straight line through the point of intersection of the lines 8x + 3y = 18, 4x + 5y = 9 and bisecting the line segment joining the points (5, -4) and (-7, 6) **[EX :5 .4(12)]**

Solution

8x + 3y = 18 ⟶ (1)

4x + 5y = 9 ⟶ (2)

2 x 2 ⟶

8x + 3y = 18

8x + 10y = 18

(-) (-) (-)

-7y = 0

y = 0

sub y = 0 in (1)

8x + 0 = 18

8x = 18

x = 18/8

x = 9/4

The point of intersection (9/4 , 0)

Midpoint of the line joining points (5, -4) and (-7, 6)

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{5-7}{2}, \frac{-4+6}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{2}{2}\right)$$

= (-1 , 1)

Equation of the line joining the points (9/4 , 0) , (-1 ,1)

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-0}{1-0} = \frac{x - 9/4}{-1 - 9/4}$$

$$\frac{y}{1} = \frac{x - 9/4}{-1 - 9/4}$$

$$\frac{y}{1} = \frac{4x - 9}{-4 - 9}$$

$$\frac{y}{1} = \frac{4x - 9}{-13}$$

4x – 9 = -13y

4x + 13y – 9 = 0

The equation of the line is 4x +13y -9 =0

For Practice

31) Find the equation of a straight line joining the point of intersection of 3x + y + 2 = 0 and x - 2y - 4 = 0 to the point of intersection of 7x - 3y = -12 and 2y = x + 3 [**EX :5 .4(11)**]

32)A mobile phone is put to use when the battery power is 100%. The percent of battery power 'y' (in decimal) remaining after using the mobile phone for x hours is assumed as y = -0.25x + 1 [**Eg:5.27**]

(i) Find the number of hours elapsed if the battery power is 40%.

(ii)How much time does it take so that the battery has no power? .

Solution

(i) To find the time when the battery power is 40% ,

y = 0.40

y = -0.25x + 1

0.40 = - 0.25x + 1

0.40 + 0.25x = 1

0.25x = 1 – 0.40

x = 0.60/ 0.25

x = $\frac{60}{25}$ = 2.4 hours

(ii) If the battery power is 0 then y = 0

0 = - 2.5x + 1

0.25x = 1

X = 1/0.25

X = 100/25

X = 4 hours.

After 4 hours, the battery of the mobile phone will have no power.

For Practice

You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by y = - 0.1x + 1

[**EX :5 .3(11)**]

(i) Find the total MB of the song..

(ii) after how many seconds will 75% of the songs gets downloaded?

(iii) After how many second the songs will be downloaded completely?

6. Trigonometry

Formula:

I

$sin\theta = \frac{\text{Opposite side}}{\text{hypothesis}}$

$\cos\theta = \frac{\text{Adjacent side}}{\text{hypothesis}}$

$tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent side}}$

$tan\theta = \frac{sin\theta}{cos\theta}$

$cosec\theta = \frac{1}{sin\theta}$

$sec\theta = \frac{1}{cos\theta}$

$cot\theta = \frac{1}{tan\theta}$

II Trigonometric ratios of complementary angles.

$\sin(90 - \theta) = \cos\theta$ $\quad$ $\cos(90 - \theta) = \sin\theta$

$\tan(90 - \theta) = \cot\theta$ $\quad$ $\cot(90 - \theta) = \tan\theta$

$\sec(90 - \theta) = \operatorname{cosec}\theta$ $\quad$ $\operatorname{cosec}(90 - \theta) = \sec\theta$

II Trigonometric Identities.

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

IV Table of trigonometric ratios for $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$

θ	0°	30°	45°	60°	90°
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cotθ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Two Marks Questions:

1) Prove that $\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

(Example 6.2)

Solution:

$$\frac{\sin A}{1+\cos A} = \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$

$$= \frac{\sin A\,(1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A\,(1-\cos A)}{1^2-\cos^2 A}$$

$$= \frac{\sin A\,(1-\cos A)}{\sin^2 A}$$

$$= \frac{1-\cos A}{\sin A}$$

2) Prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

(Example: 6.6)

Solution:

$$\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\cos\theta}}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

3) Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

(Exercise: 6.1-3(i))

Solution:

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1^2-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

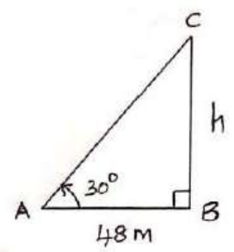
$$= \sec\theta + \tan\theta$$

Try it

4) Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

(Example: 6.5)

5) A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower. (Example: 6.19)



Solution:

Let BC be the height of the tower.

$$\tan 30° = \frac{h}{48}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{48}$$

$$\frac{h}{48} = \frac{1}{\sqrt{3}}$$

$$h = \frac{48}{\sqrt{3}}$$

$= \frac{16\times3}{\sqrt{3}}$

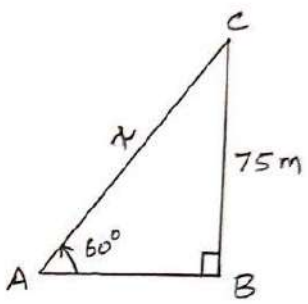
$= \frac{16\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

$= 16\sqrt{3}$

Therefore, the height of the tower is $= 16\sqrt{3}$ m.

6) A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string. (Example: 6.20)

Solution:



Let AC be the length of the string.

$\sin 60° = \frac{75}{x}$

$\frac{\sqrt{3}}{2} = \frac{75}{x}$

$x \times \sqrt{3} = 75 \times 2$

$x = \frac{150}{\sqrt{3}}$

$= \frac{50\times3}{\sqrt{3}}$

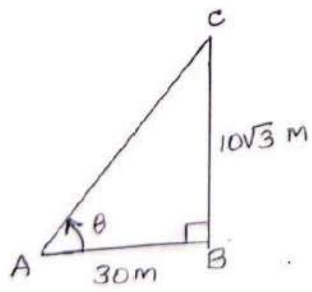
$= \frac{50\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

$= 50\sqrt{3}$

Hence, the length of the string is $= 50\sqrt{3}$ m.

7) Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m. (Exercise: 6.2 -1)

Solution:



Let BC be the height of the tower.

$\tan\theta = \frac{10\sqrt{3}}{30}$

$= \frac{\sqrt{3}}{3}$

$= \frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

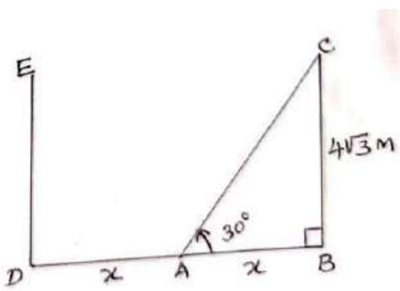
$= \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°$

8) A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}\ m$ with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road. (Exercise: 6.2 -2)

Solution:



Let BC be the height of the house.

$\tan 30° = \frac{4\sqrt{3}}{x}$

$$\frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{x}$$

$$x = 4\sqrt{3} \times \sqrt{3}$$

$$= 4 \times 3$$

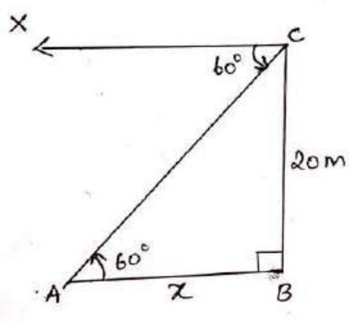
$$= 12 \text{ m}$$

Width of the road = 12 + 12

= 24 m.

9) A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3} = 1.732$) (Example: 6.26)

Solution:



$$\tan 60° = \frac{20}{x}$$

$$\sqrt{3} = \frac{20}{x}$$

$$x \times \sqrt{3} = 20$$

$$x = \frac{20}{\sqrt{3}}$$

$$= \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

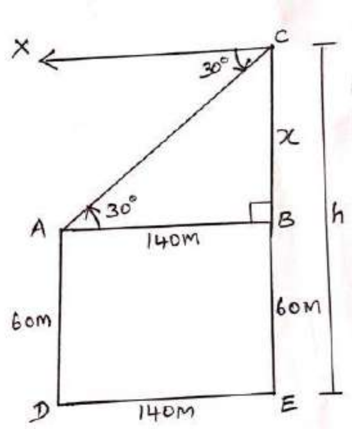
$$= \frac{20\sqrt{3}}{3} = \frac{20 \times 1.732}{3}$$

= 11.55m

10) The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30°. If the height of the first building is 60 m, find the height of the second building. ($\sqrt{3} = 1.732$)

(Example: 6.27)

Solution:



AD is the height of the first building

EC is the height of the second building

$$\tan 30° = \frac{x}{140}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{140}$$

$$\frac{140}{\sqrt{3}} = x$$

$$x = \frac{140}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{140 \times 1.732}{3}$$

$$x = 80.83$$

$$h = x + 60$$

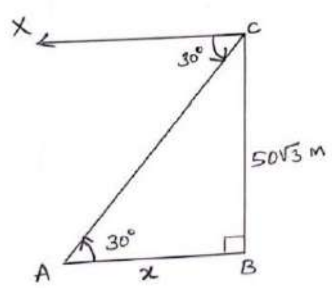
$$= 80.83 + 60$$

$$h = 140.83 \text{ m}$$

The height of the second building is 140.83 m

11) From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock. (Exercise: 6.3 - 1)

Solution:



AB is distance between car and rock

$$\tan 30° = \frac{50\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$$

$$x = 50\sqrt{3} \times \sqrt{3}$$

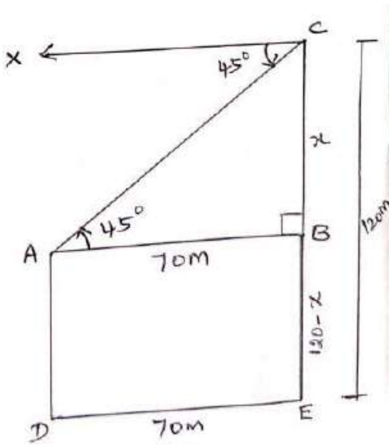
$$x = 50 \times 3$$

$$x = 150 \text{ m}$$

Distance of the car from the rock = 150 m

12) The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building. (Exercise: 6.3 - 2)

Solution:



AD is the height of the first building
CE is the height of the second building

$$\tan 45° = \frac{x}{70}$$

$$1 = \frac{x}{70}$$

$$70 = x$$

$$x = 70 \text{ m}$$

Hence, height of the first building

$$= 120 - 70$$

$$= 50 \text{ m}$$

Five Marks Questions:

1. If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ (Exercise: 6.1 – 7ii)

Solution:

$$\sqrt{3}\sin\theta - \cos\theta = 0$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

$$\text{LHS} = \tan 3\theta = \tan 3 \times 30°$$

$$= \tan 90°$$

$$= \infty$$

$$\text{RHS} = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

$$= \frac{3 * \tan 30° - \tan^3 30°}{1 - 3\tan^2 30°}$$

$$= \frac{3 \times 1/\sqrt{3} - (1/\sqrt{3})^3}{1-3 \times (1/\sqrt{3})^2}$$

$$= \frac{3/\sqrt{3} - 1/3\sqrt{3}}{1-3\times 1/3}$$

$$= \frac{3/\sqrt{3} - 1/3\sqrt{3}}{1-1}$$

$$= \frac{3/\sqrt{3} - 1/3\sqrt{3}}{0}$$

$$= \infty$$

$$\therefore \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

2. Prove that $\left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$

(Example: 6.13)

Solution:

$$= \left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right)$$

$$= \frac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \cos A \sin A)}{(\cos A - \sin A)} - \frac{(\cos A + \sin A)(\cos^2 A + \sin^2 A - \cos A \sin A)}{(\cos A + \sin A)}$$

$= (1 + \cos A \sin A) - (1 - \cos A \sin A)$

$(\because \cos^2 A + \sin^2 A = 1)$

$= 1 + \cos A \sin A - 1 + \cos A \sin A$

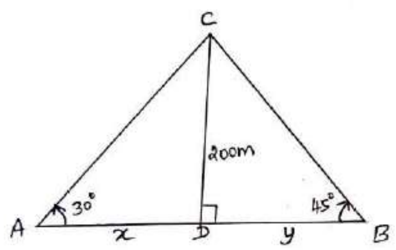
$= 2 \cos A \sin A$

$$\therefore \left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A - \sin A}\right)$$

$= 2 \cos A \sin A$

3. Two ships are sailing in the sea on either sides of a light house. The angle of elevation of the top of the light house as observed from the ships are 30° and 45° respectively. If the light house is 200 m high, find the distance between the two ships. ($\sqrt{3}$ = 1.732) (Example: 6.21)

Solution:



A, B — Positions of the two ships
CD is light house

$$\tan 30° = \frac{200}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{200}{x}$$

$x = 200\sqrt{3}$ m

$$\tan 45° = \frac{200}{y}$$

$$1 = \frac{200}{y}$$

$y = 200$ m

AB = x + y

$= 200\sqrt{3} + 200$

$= 200(\sqrt{3} + 1)$

$= 200\ (1.732+1)$

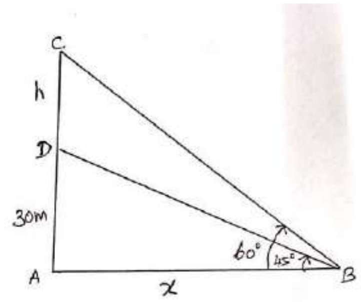
$= 200 \times 2.732$

$= 546.4$ m

Therefore, distance between two ships = 546.4 m.

4. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3}$ = 1.732) (Example 6.22)

Solution:



DC is height of the tower

$$\tan 45° = \frac{30}{x}$$

$$1 = \frac{30}{x}$$

$x = 30$ m

$$\tan 60° = \frac{30 + h}{x}$$

$$\sqrt{3} = \frac{30 + h}{x}$$

Sub x = 30, $\sqrt{3} = \frac{30 + h}{30}$

$30(\sqrt{3}) = 30 + h$

$30(\sqrt{3}) - 30 = h$

$h = 30\sqrt{3} - 30$

$h = 30\ (\sqrt{3} - 1)$

$h = 3 \times (1.732 - 1)$

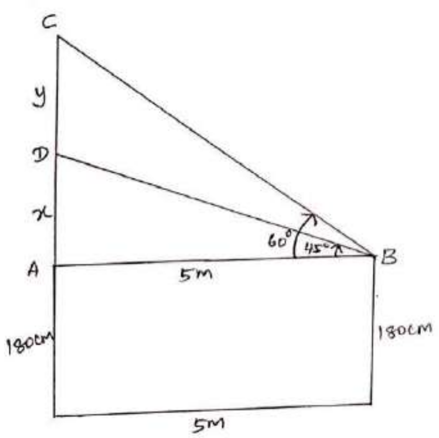
$$h = 30 \times 0.732$$

$$h = 21.96 \text{ m}$$

Hence, the height of the tower is = 21.96 m.

5. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 cm away from the wall, what is the height of the window? ($\sqrt{3}$ = 1.732) (Exericse: 6.2 – 3)

Solution:



C is top of the window

D is bottom of the window

$$\tan 45° = \frac{x}{5}$$

$$1 = \frac{x}{5}$$

$$5 = x$$

$$x = 5 \text{ m}$$

$$\tan 60° = \frac{x+y}{5}$$

$$\sqrt{3} = \frac{5+y}{5}$$

$$5\sqrt{3} = 5 + y$$

$$5\sqrt{3} - 5 = y$$

$$y = 5\sqrt{3} - 5$$

$$y = 5(\sqrt{3} - 1)$$

$$y = 5(1.732-1)$$

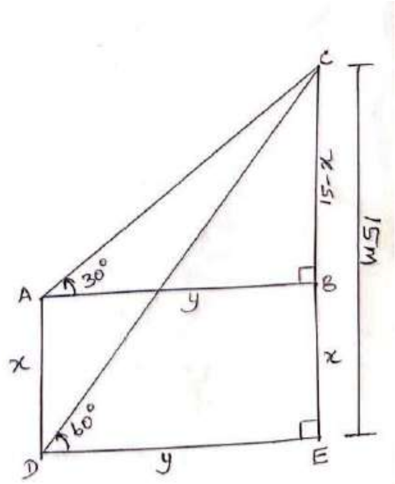
$$y = 5 \times 0.732$$

$$y = 3.66\text{m}$$

Therefore, height of window is 3.66 m

6. The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole? [Exercise 6.2 – 6]

Solution:



CE is height of the tower

AD is height of electronic pole

$$\tan 60° = \frac{15}{y}$$

$$\sqrt{3} = \frac{15}{y}$$

$$y = \frac{15}{\sqrt{3}} \quad \text{——— (1)}$$

$$\tan 30° = \frac{15-x}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{15-x}{y}$$

$$y = \sqrt{3}(15 - x) \quad \text{——— (2)}$$

From 1 & 2

$$\Rightarrow \frac{15}{\sqrt{3}} = \sqrt{3}(15 - x)$$

$$\frac{15}{\sqrt{3} \times \sqrt{3}} = 15 - x$$

$$\frac{15}{3} = 15 - x$$

$$5 = 15 - x$$

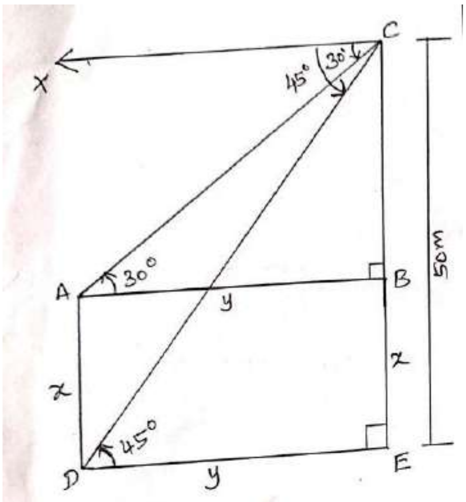
$$x = 15 - 5$$

$$x = 10 \text{ m.}$$

Hence, height of the electronic pole is = 10 m.

7. From the top of a tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be 30° and 45° respectively. Find the height of the tree. ($\sqrt{3}$ = 1.732) (Example: 6.28)

Solution:



CE is height of the tower
AD is height of the tree

$$\tan 45° = \frac{50}{y}$$

$$1 = \frac{50}{y}$$

$$y = 50 \text{ m}$$

$$\tan 30° = \frac{BC}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{BC}{50}$$

$$\frac{50}{\sqrt{3}} = BC$$

$$BC = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$BC = \frac{50 \times 1.732}{3}$$

$$BC = 28.87 \text{ m}$$

$$\therefore x = 50 - 28.87$$

$$x = 21.13 \text{ m}$$

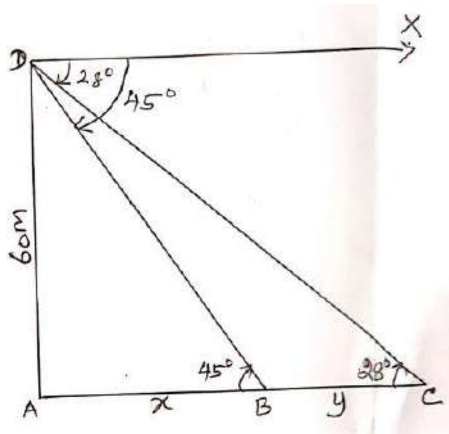
So, height of the tree is = 21.13 m.

Try it

8. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. (tan 38° = 0.7813, ($\sqrt{3}$ = 1.732) (Exercise 6.3 – 3)

9. As observed from the top of a 60 m high lighthouse from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (tan 28° = 0.5317) (Example: 6.29)

Solution:



BC — The distance between the two ships

$$\tan 45° = \frac{60}{x}$$

$$1 = \frac{60}{x}$$

$$x = 60 \text{ m}$$

$$\tan 28° = \frac{60}{AC}$$

$$0.5317 = \frac{60}{AC}$$

$$AC = \frac{60}{0.5317}$$

$$AC = 112.85 \text{ m}$$

$$x + y = 112.85$$

$$y = 112.85 - x$$

$\therefore$ y = 112.85 – 6

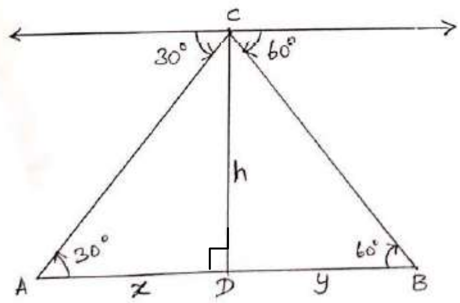
y = 52.58 m

Distance between the two ships

BC = 52.85 m

10. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $4h/\sqrt{3}$ m (Exercise 6.3 – 5)

<u>Solution:</u>



AB is the distance between the two ships

CD is lighthouse

AB = x + y

$$\tan 30° = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = h\sqrt{3}$$

$$\tan 60° = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}}$$

$$x + y = \frac{h\sqrt{3}}{1} + \frac{h}{\sqrt{3}}$$

$$= \frac{h\sqrt{3} \times \sqrt{3} + h}{\sqrt{3}}$$

$$= \frac{3h + h}{\sqrt{3}}$$

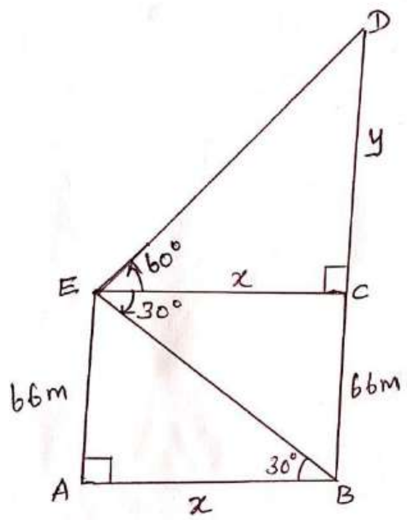
$$x + y = \frac{4h}{\sqrt{3}} \text{ m}$$

$$AB = \frac{4h}{\sqrt{3}} \text{ m}$$

$\therefore$ Distance between two ships = $\frac{4h}{\sqrt{3}}$ m.

11. From the top of a 12 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower. (Example: 6.31)

<u>Solution:</u>



AE is height of the building

BD is height of the tower

$$\tan 30° = \frac{12}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{12}{x}$$

$$x = 12\sqrt{3} \text{ m.}$$

$$\tan 60° = \frac{y}{x}$$

$$\sqrt{3} = \frac{y}{12\sqrt{3}}$$

$$\sqrt{3} \times 12\sqrt{3} = y$$

$$12 \times 3 = y$$

$$36 = y$$

$$y = 36 \text{ m.}$$

$$BD = 12 + y$$

$$= 12 + 36$$

$$BD = 48 \text{ m.}$$

Hence, Height of the tower = 48 m.

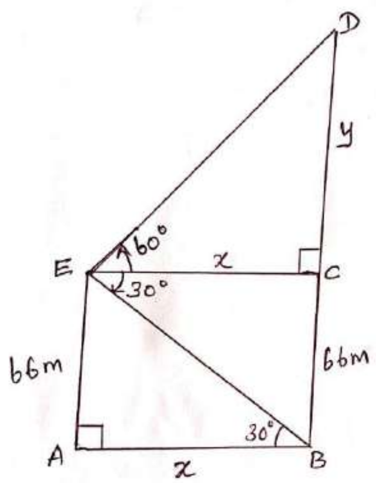
Try it.

12. From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3}$ = 1.732) (Exercise: 6.4-1)

13. The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are 60° and 30° respectively. Find

(i) The height of the lamp post.
(ii) The difference between height of the lamp post and the apartment.
(iii) The distance between the lamp post and the apartment.
($\sqrt{3}$ = 1.732) (Exercise 6.4-5)

Solution:



AE is height of the apartment
BD is height of the lamp post

$$\tan 30° = \frac{66}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{66}{x}$$

$$x = 66\sqrt{3} \text{ m}$$

$$\tan 60° = \frac{y}{x}$$

$$\sqrt{3} = \frac{y}{66\sqrt{3}}$$

$$\sqrt{3} \times 66\sqrt{3} = y$$

$$66 \times 3 = y$$

$$198 = y$$

$$y = 198 \text{ m}$$

(i) Height of the lamp post = 66 + y = 66 + 198 = 264 m

(ii) Difference between height of the lamp post and apartment = 264 – 66 = 198 m

(iii) Distance between height of the lamp post and apartment = $66\sqrt{3}$ = 66 × 1.732 = 114.31 m.

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Mensuration

Solid	CSA (Sq.units)	TSA (Sq.units)	Volume (Cu. Units)
Cylinder	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
Cone	πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemi sphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Hollow cylinder	$2\pi(R+r)h$	$2\pi(R+r)(R-r+h)$	$\pi(R^2-r^2)h$
Hollow sphere	$4\pi R^2$ = outer surface area	$4\pi(R^2+r^2)$	$\frac{4}{3}\pi(R^3-r^3)$
Hollow hemisphere	$2\pi(R^2+r^2)$	$\pi(3R^2+r^2)$	$\frac{2}{3}\pi(R^3-r^3)$
Frustum	$\pi(R+r)l$	$\pi(R+r)l+\pi r^2+\pi r^2$	$\frac{1}{3}\pi h(R^2+Rr+r^2)$

- TSA of a combined solid = C.S.A + CSA
- Volume of a combined solid = Volume + Volume
- No. of Solids = $\frac{\text{Volume of the first solid}}{\text{Volume of the second solid}}$

To find

- Radius (or) Height of the solid

  Volume = Volume

2 Marks

1. A Cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area. (Example 7.1)

Solution :

Height h = 20 cm Base radius r = 14 cm

CSA of the cylinder = $2\pi rh$ sq.units

$= 2\times\frac{22}{7}\times 14\times 20$

$= 88\times 20$

= 1760 sq.cm

TSA of the cylinder = $2\pi r(h+r)$ sq.units

$= 2\times\frac{22}{7}\times 14(20+14)$

$= 88\times 34$

= 2992 sq. cm

Try this,

A solid right circular cylinder has radius of 14 cm and height of 8 cm. Find its curved surface Area and total surface Area.

2. The curved surface area of a right circular cylinder of height 14 cm is 88 cm². Find the diameter of the cylinder. (Example 7.2)

Solution

Height h = 14 cm

CSA of the cylinder = 88 sq.unit

$2\pi rh = 88$

$2\times\frac{22}{7}\times r\times 14 = 88$

$2r = \frac{88\times 7}{22\times 14} = 2$

Diameter = 2 cm

3. **If the total surface area of a cone of radius 7cm is 704 cm², then find its slant height. (Example 7.6)**

Solution

Radius r = cm

TSA of a cone = 704 cm²

$\pi r(l+r)=704$

$\frac{22}{7}\times 7(l+7)=704$

$(l+7)=\frac{704}{22}$

$l=32-7$

= 25 cm

Slant height = 25 cm

Try this

If the CSA of a sphere is 98.56 cm², then find the radius of the sphere.

4. **Find the diameter of a sphere whose surface area is 154 m² (Example 7.8)**

Solution

Surface Area of the sphere = 154 m²

$4\pi r^2=154$

$4\times\frac{22}{7}\times r^2=154$

$r^2=\frac{154\times 7}{4\times 22}$

$r^2=\frac{7\times 7}{2\times 2}$

$r=\frac{7}{2}$

Diameter of a sphere = 2r units

$=2\times\frac{7}{2}=7m$

5. **The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases. (Example 7.9)**

Solution

Let r_1 and r_2 be the radii of the balloons.

$\frac{r_1}{r_2}=\frac{12}{16}=\frac{3}{4}$

ratio of CSA of balloons = $4\pi r_1^2 : 4\pi r_2^2$

$=\frac{4\pi r_1^2}{4\pi r_2^2}=\frac{r_1^2}{r_2^2}$

$=\left(\frac{r_1}{r_2}\right)^2=\left(\frac{3}{4}\right)^2=\frac{9}{16}$

= 9 : 16

6. **If the base area of a hemispherical solid is 1386 sq.m, then find its total surface area? (Example 7.10)**

Solution : Given that,

Base area = 1386 sq.m

$\pi r^2=1386$ sq.m

TSA of a hemishere = $3\pi r^2$ sq.m

= 3 × 1386

= 4158 m²

7. **The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area. (Example 7.13)**

Solution : Given that

l = 5cm, R = 4 cm, r = 1 cm

CSA of the frustum = $\pi(R+r)l$ sq.units

$=\frac{22}{7}\times(4+1)\times 5$

$=\frac{22}{7}\times 5\times 5$

$=\frac{550}{7}$

= 78.57 cm²

8. **Find the volume of a cylinder whose height is 2m and whose base area is 250m^2 (Example 7.15)**

Solution : Given that

Height h = 2m

Base area = 250 m^2

$$\pi r^2 = 250$$

Volume of a cylinder = $\pi r^2 h$ Cu.units

= 250 × 2

= 500 m^3

9. **The volume of a solid right circular cone is 11088cm^3. If its height is 24cm then find the radius of the cone. (Example 7.19)**

Solution : Given that

Height h = 24 cm

Volume of the cone = 11088 cm^3

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$r^2 = \frac{11088 \times 3 \times 7}{22 \times 24}$$

$$r^2 = 441$$

$$r = \sqrt{441}$$

r = 21

radius of the cone = 21 cm

Try this:

The volume of a cone is 4928cm^3. If its height is 24 cm then find the radius of the cone.

10. **If the circumference of a conical wooden piece is 484 cm, then find its volume when its height is 105cm. (Exercise 7.2 sum 3)**

Solution

Circumference of a cone = 484 cm

$$2\pi r = 484$$

$$2 \times \frac{22}{7} \times r = 484$$

$$r = \frac{484 \times 7}{2 \times 22}$$

$$= 11 \times 7$$

r=77 cm

Volume of a cone = $\frac{1}{3}\pi r^2 h$ cu.units

$$= \frac{1}{3} \times \frac{22}{7} \times 77 \times 77 \times 105$$

$$= 22 \times 11 \times 77 \times 35$$

= 652190 cm^3

11. **The volumes of two cones of same base radius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights. (Exercise 7.2 sum 6)**

Solution.

Given radius = $r_1 = r_2$

Ratio of volumes of 2 cones = $\frac{3600}{5040}$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3600}{5040}$$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_1^2 h_2} = \frac{3600}{5040} \quad (\because r_1 = r_2)$$

$$\frac{h_1}{h_2} = \frac{3600}{5040}$$

$$\frac{h_1}{h_2} = \frac{5}{7}$$

$$h_1 : h_2 = 5 : 7$$

Ratio of heights = 5:7

12. **If the ratio of radii of two spheres is 4:7, find the ratio of their volumes (Exercise 7.2 sum 7)**

Solution.

Let r_1, r_2 be the radii of two sphere.

Given that $r_1 = 4$ $r_2 = 7$

Ratio of their volumes = $\frac{4}{3}\pi r_1^3 : \frac{4}{3}\pi r_2^3$

$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$= \frac{r_1^3}{r_2^3}$$

$$= \frac{4^3}{7^3} = \frac{64}{343}$$

Ratio of their volumes = 64 : 343

5 Marks

1. **A garden roller whose length is 3m long and whose diameter is 2.8m is rolled to level a garden. How much area will it cover in 8 revolutions? (Example 7.3)**

Solution : Given that

diameter = 2.8 m

radius r = $\frac{2.8}{2} = 1.4$ m

height h = 3m

Area covered in one revolution = CSA of a cylinder

$= 2\pi rh$ sq.units

$= 2 \times \frac{22}{7} \times 1.4 \times 3$

= 26.4 m^2

Area covered in 8 revalutions = 8 × 26.4

= 211.2m^2

2. **If the radii of the circular ends of a frustum which is 45 cm high are 28cm and 7 cm, find the volume of the frustum. (Example 7.23)**

Solution : Given that

Height h = 45 cm

top radius R = 28 cm

bottom radius r = 7 cm

volume of the frustum

$= \frac{1}{3}\pi h\left[R^2 + Rr + r^2\right]$ cu.units

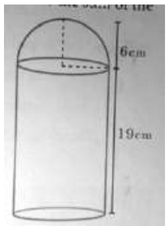
$= \frac{1}{3} \times \frac{22}{7} \times 45\left[28^2 + (28 \times 7) + 7^2\right]$

$= \frac{1}{3} \times \frac{22}{7} \times 45 \times 1029$

= 48510 cm^3

3. **A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25cm. Find the total surface area of the toy if its common diameter is 12cm. (Example 7.24)**

Solution



Cylinder

Diameter (d) = 12 cm,

Radius (r) = $\frac{12}{2} = 6$ cm,

Height (h) = 25-6= 19 cm

hemisphere

Diameter = 12 cm, radius r = 6 cm

TSA of the toy = CSA of the cylinder + CSA of the hemisphere + Base area of the cylinder.

$= 2\pi rh + 2\pi r^2 + \pi r^2$

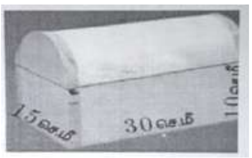
$= \pi r(2h + 3r)$

$= \frac{22}{7} \times 6(38 + 18)$

$= \frac{22}{7} \times 6 \times 56$

= 1056 cm^2

4. **A jewel box is in the shape of a cuboid of dimensions 30cm×15cm×10cm surmounted by a half part of a cylinder. Find the volume of the box. (Example 7.25)**



Solution

Cuboid

length (l) = 30 cm, breadth (b) = 15cm, height (h) = 10 cm

Cylinder

diameter = 15 cm

Radius (r) = $\frac{15}{2}$ cm

Height h_1 = 30 cm

Volume of the box=(Volume of the cuboid)+ $\frac{1}{2}$ (Volume of the cylinder)

$$= (l \times b \times h) + \left[\frac{1}{2}\pi r^2 h_1\right]$$

$$= (30\times15\times10) + \left[\frac{1}{2}\times\frac{22}{7}\times\frac{15}{2}\times\frac{15}{2}\times30\right]$$

$$= 4500 + \left[\frac{11\times15\times15\times15}{7\times2}\right]$$

$$= 4500 + \left[\frac{165\times225}{14}\right]$$

$$= 4500 + \left[\frac{37125}{14}\right]$$

$$= 4500 + 2651.785$$

$$= 7151.79 \text{ cm}^3$$

5. **A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14cm and the height of the vessel is 13cm. Find the capacity of the vessel. (Exercise 7.3 sum 1)**

Solution

Hemisphere

Diameter = 14 cm, radius r = $\frac{14}{2}$ = 7 cm

Cylinder

radius r = 7 cm

Height h = 13 − 7 = 6 cm

Capacity of the vessel = Volume of the hemisphere + Volume of the cylinder

$$= \frac{2}{3}\pi r^3 + \pi r^2 h \text{ cu.unit}$$

$$= \pi r^2\left[\frac{2}{3}r + h\right]$$

$$= \frac{22}{7}\times7\times7\left[\frac{2}{3}(7)+6\right]$$

$$= 154\times\frac{32}{3} = \frac{4928}{3}$$

$$= 1642.67 \text{ cm}^3$$

6. **Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made. (Exercise 7.3 sum 2)**

Solution

Cylinder

Diameter= 3 cm, radius r = $\frac{3}{2}$ cm

Height h_1 = 12 − 4 = 8 cm

Cone

Diameter= 3 cm

Radius r = $\frac{3}{2}$ cm

Height h_2 = 2 cm

Volume of the model = Volume of the cylinder + 2 volume of the cones

$$= \pi r^2 h_1 + 2\times\frac{1}{3}\pi r^2 h_2$$

$$= \pi r^2\left[h_1 + \frac{2}{3}h_2\right]$$

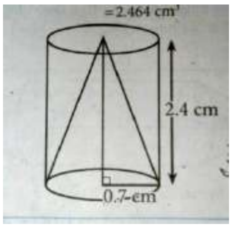
$$= \frac{22}{7}\times\left(\frac{3}{2}\right)^2\left[8+\frac{4}{3}\right]$$

$$= \frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\times\frac{28}{3}$$

$$= 66 \text{ cm}^3$$

7. **From a solid cylinder whose height is 2.4cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm³ (Exercise 7.3 Sum 3)**

Solution



Cylinder

Diameter = 1.4 cm

Radius r = 0.7 cm

Height h = 2.4 cm

Cone

Diameter = 1.4 cm

Radius r = 0.7 cm

Height h = 2.4 cm

Volume of the remaining solid = Volume of the cylinder - Volume of the cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \pi r^2 h(1-\frac{1}{3})$$

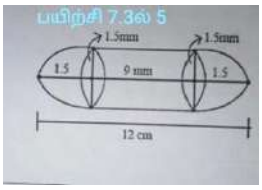
$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3}\times\frac{22}{7}\times 0.7\times 0.7\times 2.4$$

$$= 2.464 \text{ cm}^3$$

8. **A capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how mach medicine it can hold? (Example 7.3 sum 5)**

Solution



Cylinder

Diameter = 3 mm

Radius r = $\frac{3}{2}$ mm

Height h = (12-3) = 9mm

hemisphere

Diameter = 3mm

Radius = $\frac{3}{2}$ mm

Volume of capsule = Volume of the cylinder + Volume of 2 hemispheres

$$= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)$$

$$= \pi r^2\left[h + \frac{4}{3}r\right]$$

$$= \frac{22}{7}\times\frac{9}{4}\times\left[9+\left(\frac{4}{3}\times\frac{3}{2}\right)\right]$$

$$= \frac{11}{7}\times\frac{9}{2}\times 11$$

$$= 77.79 \text{mm}^3$$

9. **A metallic spheres of radius 16 cm is melted and recast into small sphere each of radius 2cm. How many small spheres can be obtained? (Example 7.29)**

Solution

Big sphere

Radius R = 16 cm

Small sphere

Radius r = 2 cm

No. of small spheres = $\frac{\text{Volume of big sphere}}{\text{Volume of small sphere}}$

$$n = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$$

$$= \frac{\frac{4}{3}\pi \times (16)^3}{\frac{4}{3}\pi \times (2)^3}$$

$$= \frac{16\times16\times16}{2\times2\times2}$$

= 512 small spheres.

10. **A cone of height 24cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder. (Example : 7.30)**

Solution

Cone

height h_1 = 24 cm

radius = r cm

Cylinder

height h_2 = ?

radius = r cm

Volume of cylinder = Volume of cone

$$\pi r^2 h_2 = \frac{1}{3}\pi r^2 h_1$$

$$h_2 = \frac{1}{3}h_1$$

$$= \frac{1}{3}\times 24$$

=8

Height of cylinder = 8 cm

11. **An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder. (Exercise7.4 sum 1)**

Solution

Sphere

Radius r_1 = 12 cm

Cylinder

Radius r_2 = 8 cm

Height h = ?

Volume of the cylinder = Volume of sphere

$$\pi r_2^2 h = \frac{4}{3}\pi r_1^3$$

$$8^2 h = \frac{4}{3}\times 12^3$$

$$8\times8\times h = \frac{4}{3}\times12\times12\times12$$

$$h = \frac{4\times12\times12\times12}{3\times8\times8}$$

= 36 cm

Height of the cylinder = 36 cm

12. **A right circular cylindrical container of base radius 6 cm and height 15cm is full of ice cream. The ice cream is to be filled in cones of height 9cm and base radius 3cm, having a hemispherical cap. Find the number of cones needed to empty the container. (Example 7.31)**

Solution

Cylinder

Radius r_1 = 6 cm

Height h_1 = 15 cm

Cone

Radius r_2 = 3 cm

Height h_2 = 9 cm

Hemisphere

Radius $r_2 = 3$ cm

No. of cones = $\frac{\text{Volume of cylinder}}{\text{Volume of cone + Volume of the hemisphere}}$

$$= \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3}$$

$$= \frac{\pi \times 6^2 \times 15}{\frac{1}{3}\pi r_2^2 [h_2 + 2r]}$$

$$= \frac{\pi \times 6 \times 6 \times 15 \times 3}{\pi \times 3 \times 3 [9 + 2(3)]}$$

$$= \frac{2 \times 6 \times 15}{15}$$

= 12 ice cream cones.

13. A solid right circular cone of diameter 14cm and height 8cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm. Find the internal diameter. (Exercise 7.4 sum 4)

Solution

Cone

Diameter = 14 cm

radius r = 7 cm

Height h = 8 cm

Hollow sphere

external Diameter = 10 cm

external radius R = 5 cm

Internal Radius r = ?

Internal Diameter = ?

Volume of hollow sphere= Volume of the cone.

$$\frac{4}{3}\pi[R^3 - r^3] = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3}\pi[5^3 - r^3] = \frac{1}{3}\pi \times 7^2 \times 8$$

$$4[5^3 - r^3] = 7 \times 7 \times 8$$

$$5^3 - r^3 = \frac{7 \times 7 \times 8}{4}$$

$$125 - r^3 = 98$$

$$r^3 = 27$$

$$r = 3 \text{ cm}$$

Internal Diameterd = 2 r Units

= 2 × 3 = 6 cm

14. Find the number of coins, 1.5cm in diameter and 2 mm thick to be melted to form a right circular cylinder of height 10 cm and diameter 4.5cm. (Unit Exercise 7 sum 5)

Solution

Cylinder

Height $h_1 = 10$ cm

Diameter = 4.5 cm

Radius $r_1 = 2.25$ cm

Coins (cylindrical)

Diameter = 1.5 cm

Radius $r_2 = 0.75$ cm

Height $h_2 = 2$ mm

$$= \frac{2}{10} = 0.2 \text{ cm}$$

Number of coins = $\frac{\text{Volume of the cylinder}}{\text{Volume of one coin}}$

$$= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

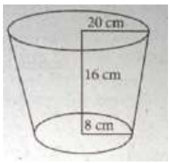
$$= \left(\frac{2.25}{0.75}\right)^2 \times \frac{10}{0.2}$$

$$= 3^2 \times 50$$

= 450 coins.

Try these

15. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹ 40 per litre.



(Exercise 7.2 sum 10)

Solution.

Given radius of lower end r = 8 cm

Radius of upper end R = 20 cm

Height h = 16 cm

Volume of the frustum= $\frac{\pi h}{3}(R^2+r^2+Rr)$ cu.units

$= \frac{22\times16}{7\times3}\left[(20)^2+(8)^2+20\times8\right]$

$= \frac{352}{21}[400+64+160]$

$= \frac{352\times624}{21}$

$= \frac{219648}{21} = 10459.4\ cm^3$

$= \frac{10459.4}{1000}$ [∴ 1000 cm^3 = 1 litre]

= 10.4594 litre.

Cost of milk per litre = Rs. 40.

∴ Total cost = 10.459 × 40

= Rs. 418.36

16. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is Rs. 2. (Exercise7.1 sum 10)

Solution

From the figure r = 6 cm

R = 12 cm

h = 8 cm

$l = \sqrt{h^2+(R-r)^2} = \sqrt{8^2+(12-6)^2} = \sqrt{8^2+6^2}$

$= \sqrt{64+36} = \sqrt{100}$

l = 10 cm

Area to be painted = CSA of the frustum+ area of top circular region

$= \pi l(R+r)+\pi r^2$

$= \pi\left[l(R+r)+r^2\right]$

$= \frac{22}{7}\times\left[10(12+6)+6^2\right]$

$= \frac{22}{7}\times[180+36]$

$= \frac{22}{7}\times216$

$= \frac{4752}{7} \simeq 678.86$

Cost of painting per sq.cm = Rs.2

∴ Total cost = 678.86 × 2= Rs. 1357.72

17. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm (Exercise 7.1 sum 6)

Solution

Area of the paper = 5720 cm²

Given radius of birthday cap r = 5 cm

Height of birthday cap h = 12 cm

∴ Slant height $l = \sqrt{h^2+r^2}$

$= \sqrt{12^2+5^2}$

$= \sqrt{144+25}$

$= \sqrt{169}$

$= 13$ cm

CSA of conical cap = $\pi r l$ sq.units

$= \frac{22}{7} \times 5 \times 13$

$= \frac{1430}{7}$

∴ Number of birthday caps =

$\frac{\text{Area of paper sheet}}{\text{CSA of conical cap}}$

$= \frac{5720}{1430} \times 7$

$= 28$ caps.

STATISTICS

1.Range = L-S

2.C0-efficient of Range = $\frac{L-S}{L+S}$

3.Standard deviation of first 'n' natural numbers , $\sigma = \sqrt{\frac{n^2-1}{12}}$

3.Standard deviation (Ungrouped data) , $\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$

4.Standard deviation (grouped data), $\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$

5.When multiply or divide each data by some constant k , the standard deviation is also multiply or divide by k.

6.When increased or decreased each data by some constant k , the standard deviation will not change

7.coefficient of Variation , C.V = $\frac{\sigma}{\bar{x}} \times 100$, $\sigma = \sqrt{\frac{\sum d^2}{n}}$

PROBABILITY

1. $P(E) = \frac{n(E)}{n(S)}$

2. Tossing an coin twice S = {HH,HT,TH,TT}, n(S)=4

3. Tossing an coin thrice S={ HHH , HHT , HTH , HTT , THH, THT , TTH , TTT }, n(S)=8

4. Rolling a die once S ={1,2,3,4,5,6}, n(S)=6

5. Rolling a dice twice S = {(1,1) ………………..(6,6)} ,n(S)=36

6.P(AUB)=P(A)+P(B)- $P(A \cap B)$

7.If A and B are mutually exclusive events P(AUB)=P(A)+P(B)

8.$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

9. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

10. $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

11. $P(\bar{A}) = 1 - P(A)$

12 . No. of cards =52

13.No.of black card =26

14.No.of red card =26

15. No.of red king =2

16.No.of black Queen =2

17.No.of clavor =13

18.no.of heart card =13

19. No.of face card =12

20. No.of number card =36

2 Mark Questions

1) (i)Find the range and coefficient of range of 63,89,98,125,79,108,117,68 (Exercise8.1 (1))

Solution:
range = L-S = 125-63=62

coefficient of range = $\frac{L-S}{L+S}=\frac{125-63}{125+63}=\frac{62}{188}=0.33$

***FOR PRACTICE*:**

2) Find the range and coefficient of range of 25,67,48,53,18,39,44 (Example (8.1))

3) Find the range and coefficient of range of 43.5,13.6,18.9,38.4,61.4,29.8 (Exercise8.1 (1))

4) If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the Large value. (Exercise8.1 (2))

Solution:
L=R+S = 36.8+13.4=50.2

FOR PRACTICE

5) If the range of a set of data is 13.67 *and* the largest value is 70.08 Find the Smallest Value. (Example 8.3)

6) Calculate the range of the following data (Exercise8.1 (3))

Income	400-450	450-500	500-550	550-600	600-650
Number of Workers	8	12	30	21	6

Solution:
R=L-S =650-400=250

FOR PRACTICE

7) Example 8.2

Find the Range

Age	16-18	18-20	20-22	22-24	24-26	26-28
Number of Students	0	4	6	8	2	2

8) If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new Standard deviation. (Exercise8.1 (8))

Solution:
The Standard deviation will not change when we add some value to all the values . The new Standard deviation is 4.5

9) If the standard deviation of a data is 3.6 and if each value of the data is divided by 3, then find the new variance and new Standard deviation. (Exercise8.1 (9)) **Solution:**
when we divide each value by 3 then the standard deviation also divided by 3

New standard deviation , σ =3.6 /3 =1.2
Variance , σ^2 =1.2 X 1.2 =1.44

10) Find the Standard deviation of First 21 natural numbers. (Exercise8.1 (7))

$$\sigma=\sqrt{\frac{n^2-1}{12}}$$
$$=\sqrt{\frac{21^2-1}{12}}=\sqrt{\frac{440}{12}}=\sqrt{36.6}=6.05$$

11) The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the Coefficient of variation . (Exercise8.2 (1))

Solution:

$$c.v=\frac{\sigma}{\bar{x}}\times 100=\frac{6.5}{12.5}\times 100=52\%$$

12) The standard deviation and Coefficient of variation of a data are 1.2 and 25.6 respectively.Find the value of mean. (Exercise8.2 (2))

Solution:

$$c.v=\frac{\sigma}{\bar{x}}\times 100$$
$$25.6=\frac{1.2}{\bar{x}}\times 100$$
$$\bar{x}=\frac{1.2}{25.6}\times 100$$
$$\bar{x}=4.96$$

13) The mean and Coefficient of variation of a data are 15 and 48 respectively.Find the value of standard deviation. (Exercise8.2 (3))

Solution:

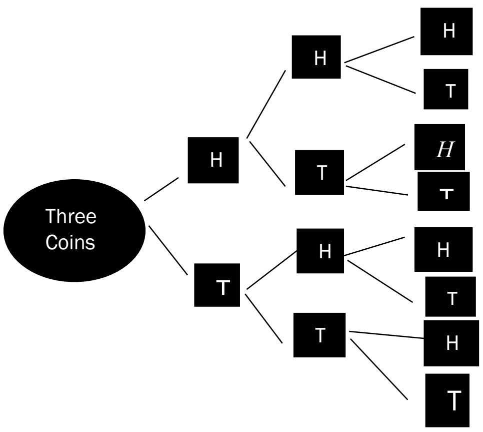
$c.v = \frac{\sigma}{x} \times 100$

$48 = \frac{\sigma}{15} \times 100$

$\sigma = \frac{48 \times 15}{100}$

$\sigma = 7.2$

14) Write the sample space for tossing three coins using tree diagram.(Exercise8.3 (1))



S={ HHH , HHT , HTH , HTT , THH, THT , TTH , TTT }

FOR PRACTICE

15) Write the sample space for selecting two balls from at a time from a bag containing 6 balls numbered I to 6 (Using Tree diagram) (Exercise8.3 (2))

16) Express the sample space for rolling two dice using tree diagram (Example 8.17)

17)Two coins are tossed together .What is the probability of getting different faces on the coins ? (Example 8.20)

Solution:

S={HH,HT,TH,TT} n(s) =4

A={HT , TH} n(A)=2

P(A)= $\frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

18) A Coin is tossed thrice .What is the Probability of getting two concecutive tails? (Exercise 8.3(4))

Solution:

S={ HHH,HHT,HTH ,HTT ,THH,THT ,TTH ,TTT }

n(S)=8

A={ HTT, TTH, TTT }

n(A)=3

P(A)= $\frac{n(A)}{n(S)} = \frac{3}{8}$

19) A bag contains 5 blue balls and 4 green balls .A ball is drawn at random from the bag. Find the Probability that thye ball drawn is (i) blue (ii) not blue (Example 8.18)

Solution:

n(S)=5+4=9

(i) n(A)=5 P(A)= $\frac{n(A)}{n(S)} = \frac{5}{9}$

(ii) n(B)=4 P(B)= $\frac{n(B)}{n(S)} = \frac{4}{9}$

20) If A is an event of a random experiment such that $P(A):P(\overline{A}) = 17:15$ and n(S)=640 then find.

$(i)P(\overline{A})$ $(ii)n(A)$ (Exercise8.3 (3))

Solution:

$\frac{P(A)}{P(\overline{A})} = \frac{17}{15}$

$\frac{1-P(\overline{A})}{P(\overline{A})} = \frac{17}{15}$

$15(1-P(\overline{A})) = 17P(\overline{A})$

$15 = 32P(\overline{A})$

$P(\overline{A}) = \frac{15}{32}$

$P(A)+P(\overline{A}) = 1$

$P(A)+\frac{15}{32} = 1$

$P(A) = 1-\frac{15}{32} = \frac{17}{32}$

$\frac{n(A)}{n(S)} = \frac{17}{32}$

$\frac{n(A)}{640} = \frac{17}{32}$

$n(A) = \frac{17}{32} \times 640 = 340$

21) If $P(A) = \frac{2}{3}, P(B) = \frac{2}{5}, P(AUB) = \frac{1}{3}$ then find $P(A \cap B)$ (Exercise 8.4 (1))

Solution:

$P(A \cup B) = P(A)+P(B)-P(A \cap B)$

$\frac{1}{3} = \frac{2}{3}+\frac{2}{5}-P(A \cap B)$

$P(A \cap B) = \frac{2}{3}+\frac{2}{5}-\frac{1}{3}$

$P(A \cap B) = \frac{16}{15}-\frac{1}{3} = \frac{11}{15}$

For practice

22) $P(A) = 0.37$ $P(B) = 0.42$ $P(A \cap B) = 0.09$ *then find* $P(A \cup B)$

(Example 8.26)

23) A and B are two events such that $P(A)=0.42$ $P(B)=0.48$ *and* $P(A\cap B)=0.16$ Find $P(not\ A)$ (ii) $P(not\ B)$ (iii) $P(A\ or\ B)$ (Exercise 8.4 (2))

Solution:

$P(A)=0.42 \quad P(B)=0.48 \quad P(A\cap B)=0.16$

$(i) P(not\ A)=P(\overline{A})=1-P(A)=1-0.42=0.58$

$(ii) P(not\ B)=P(\overline{B})=1-P(B)=1-0.48=0.52$

$(iii) P(A or\ B)=P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.42+0.48-0.16=0.74$

---

24) If A and B are two mutually exclusive events of a random experiment and $P(not\ A)$ =0.45, $P(A\cup B)=0.65$ *then find* P(B) (Exercise 8.4 (3))

Solution:

$P(\overline{A})=0.45 \quad P(A)=1-P(\overline{A})=1-0.45=0.55$

$P(A\cup B)=P(A)+P(B)$

$0.65=0.55+P(B)$

$P(B)=0.1$

---

25) What is the Probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards.(Example 8.27)

Solution:

$n(S)=52$

$P(A)=\frac{4}{52}, \qquad P(B)=\frac{4}{52}$

$P(AUB)=P(A)+P(B)=\frac{4}{52}+\frac{4}{52}=\frac{8}{52}=\frac{2}{13}$

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5 Mark Questions

1) A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day.Find the standard deviation of the number of strikes the bell make a day.(Exercise 8.1 (6))

Solution:

Number of strikes the bell make a day

=2(1+2+3+4+5+6+7+8+9+10+11+12)

$=2\left(\frac{n(n+1)}{2}\right)=2\left(\frac{12\times 13}{2}\right)=2\times 78=156$

standard deviation

$\sigma=2\sqrt{\frac{n^2-1}{12}}$

$=2\sqrt{\frac{12^2-1}{12}}=\sqrt{\frac{143}{12}}=\sqrt{11.92}=6.90$

---

2) Find the variance and standard deviation of the wages of 9 workers given below:

₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280. (Exercise8.1 (5))

Soltuion:

A= 300

x	$d=x-A$	d^2
280	-20	400
280	-20	400
290	-10	100
290	-10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
	$\sum d$ =0	$\sum d^2$ =2000

$$\sigma=\sqrt{\frac{\sum d^2}{n}-\left(\frac{\sum d}{n}\right)^2}$$

$$=\sqrt{\left(\frac{2000}{9}-\left(\frac{0}{5}\right)^2\right)}$$

$$=\sqrt{222.22}=14.91$$

variance=222.22

standard deviation = 14.91

---

3) A teacher asked the students to complete 60 pages of a record note book .Eight students have completed only 32,35,37,30,33,36,35,37 pages . Find the standard deviation of the pages completed by them.(Exercise8.1 (4))

Solution:

A= 35

x	$d=x-A$	d^2
32	-3	9
35	0	0
37	2	4
30	-5	25
33	-2	4
36	1	1
35	0	0
37	2	4
	$\sum d$ =-5	$\sum d^2$ =47

$$\sigma=\sqrt{\frac{\sum d^2}{n}-\left(\frac{\sum d}{n}\right)^2}$$

$$=\sqrt{\left(\frac{47}{8}-\left(\frac{-5}{8}\right)^2\right)}$$

$$=\sqrt{\frac{47}{8}-\frac{25}{64}}=\sqrt{\frac{376-25}{64}}=\sqrt{\frac{351}{64}}=\sqrt{5.48}=2.34$$

FOR PRACTICE

4) The number of televisions sold in each day of a week are 13,8,4,9,7,12,10. Find its standard deviation .(Example 8.4)

5) The amount of rain fall in a particular season for six days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm,12.8,11.4 cm. Find its standard deviation.(Example 8.5)

6) The marks scored by 10 students in a class test are 25,29,30,33,35,37,38,40,44,48 . Find the standard deviation.(Example 8.6)

7) The amount that the children have spent for purchasing some eatables in one day trip of a school are 5,10,15,20,25,30,35,40 .Find the standard deviation of the amount they have spent. (Example 8.7)

8) The rainfall recorded in various places of five districts in a week are given below. Find its standard deviation.(Exercise8.1 (10))

Rainfall	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Solution:

x	f	d=x-A d=x-60	fd	fd^2
45	5	-15	-75	1125
50	13	-10	-130	1300
55	4	-5	-20	100
60	9	0	0	0
65	5	5	25	125
70	4	10	40	400
	N=40		∑fd=-160	∑fd^2=3050

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= \sqrt{\frac{3050}{40} - \left(\frac{-160}{40}\right)^2}$$

$$= \sqrt{76.25 - 16}$$

$$= \sqrt{60.25}$$

$$= 7.76$$

FOR PRACTICE:

9). 48 students were asked to write the total number of hours per week they spent on watching television.With this information find the standard deviation of hours spent for watching television.(Example 8.11)

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

10) The marks Scored by the students in a slip test are given below . Find the standard deviation of their marks.(Example 8.12)

x	4	6	8	10	12
f	7	3	5	9	5

11) In a study about viral fever, the number of people affected in a town were noted as. Find its standard deviation.(Exercise8.1 (11))

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of affected peoples	3	5	16	18	12	7	4

Solution:

A =35

Age	Mid value x	f	d=x-A d=x-35	fd	fd^2
0-10	5	3	-30	-90	2700
10-20	15	5	-20	-100	2000
20-30	25	16	-10	-160	1600
30-40	35	18	0	0	0
40-50	45	12	10	120	1200
50-60	55	7	20	140	2800
60-70	65	4	30	120	3600
		N=65		∑fd=30	∑fd^2=13900

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= \sqrt{\frac{13900}{65} - \left(\frac{30}{65}\right)^2}$$

$$= \sqrt{\frac{13900}{65} - \frac{900}{4225}}$$

$$= \sqrt{213.85 - 0.21}$$

$$= \sqrt{213.64}$$

$$= 14.62$$

FOR PRACTICE

12) Marks of the students in a particular subject of a class are given below. Find its standard deviation.(Example 8.13)

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	8	12	17	14	9	7	4

13) Find the coefficient of variation of 24,26,33,37,29,31.(Exercise 8.2 (5))

Solution:

$\bar{x} = 30$

X	d =x- $\bar{x}$	d^2
24	-6	36
26	-4	16
29	-1	1
31	1	1
33	3	9
37	7	49
		$\sum d^2 = 112$

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{112}{6}} = 4.32$$

$$c.v = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.32}{30} \times 100 = 14.4\%$$

14) The time taken to complete a homework by students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find its coefficient of variation. (Exercise 8.2 (6))

Solution:

.

$\bar{x} = 45$

x	d =x- $\bar{x}$	d^2
38	-7	49
40	-5	25
43	-2	4
44	-1	1
46	1	1
47	2	4
49	4	16
53	8	64
		$\sum d^2 = 164$

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{164}{8}} = 4.53$$

$$c.v = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.53}{45} \times 100 = 10.07\%$$

15) Two unbiased dice are rolled once .Find the Probability of getting (i) a doublet (ii) the product as a prime number (iii) the sum as a prime number (iv) the sum as 1.(Exercise8.3 (7))

Solution:

$$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$$
$$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$$
$$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$$
$$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$$
$$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$$
$$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$$

$n(S) = 36$

$(i) A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$(ii) B = \{(1,2),(2,1),(1,3),(3,1),(1,5),(5,1)\}$

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$(iii) C = \{(1,1),(1,2),(2,1),(1,4),(2,3),(3,2),(4,1)$
$(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(5,6),(6,5)\}$

$n(C) = 15$

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

$(iv) \quad D = \{\ \}$

$n(D) = 0$

$$P(D) = \frac{n(D)}{n(S)} = \frac{0}{36} = 0$$

16) Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday).Each is equally likely to visit the shop on any one day as on another day.What is the Probability that both will visit the shop on (i) the same day (ii) different days (iii) consecutive days? (Exercise8.3 (13))

Solution:

$S = \{(MON,MON)(MON,TUE)(MON,WED)(MON,THU)(MON,FRI)(MON,SAT)$
$(TUE,MON)(TUE,TUE)(TUE,WED)(TUE,THU)(TUE,FRI)(TUE,SAT)$
$(WED,MON)(WED,TUE)(WED,WED)(WED,THU)(WED,FRI)(WED,SAT)$
$(THU,MON)(THU,TUE)(THU,WED)(THU,THU)(THU,FRI)(THU,SAT)$
$(FRI,MON)(FRI,TUE)(FRI,WED)(FRI,THU)(FRI,FRI)(FRI,SAT)$
$(SAT,MON)(SAT,TUE)(SAT,WED)(SAT,THU)(SAT,FRI)(SAT,SAT)\}$

$n(S) = 36$

$(i) A=\{(MON,MON),(TUE,TUE),(WED,WED),(THU,THU),(FRI,FRI),(SAT,SAT)\}$

$n(A)=6$

$P(A)=\frac{n(A)}{n(S)}=\frac{6}{36}=\frac{1}{6}$

$(ii) P(\overline{A})=1-P(A)=1-\frac{1}{6}=\frac{5}{6}$

$(iii) C=\{(MON,TUE),(TUE,MON),(TUE,WED),(WED,TUE),(WED,THU),(THU,WED)$
$(THU,FRI),(FRI,THU),(FRI,SAT),(SAT,FRI)\}$

$n(C)=10$

$P(C)=\frac{n(C)}{n(S)}=\frac{10}{36}=\frac{5}{18}$

FOR PRACTICE

17) Two dice are rolled .Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13.(Example 8.19)

18) Two dice are rolled once.Find the probability of getting an even numberon the first die or a total of face sum 8.(Exercise8.4 (6))

$$S=\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$$
$$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$$
$$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$$
$$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$$
$$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$$
$$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$$

$n(S)=36$

$$A=\{(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$$
$$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$$
$$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)\}$$

$$P(A)=\frac{n(A)}{n(S)}=\frac{18}{36}=\frac{1}{2}$$

$$B=\{(2,6)(3,5)(4,4)(5,3)(6,2)\}$$

$$P(B)=\frac{n(B)}{n(S)}=\frac{5}{36}$$

$$A\cap B=\{(2,6)(4,4)(6,2)\}$$

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{3}{36}$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=\frac{18}{36}+\frac{5}{36}-\frac{3}{36}=\frac{20}{36}=\frac{5}{9}$$

FOR PRACTICE

19) Two dice are rolled together.Find the probability of getting a doublet or sum of faces as 4.
(Example 8.28)

20) From a well shuffled pack of 52 cards . one card is drawn at random . Find the probability of getting(i) red card (ii) heart card (iii) red king (iv) face card (v) number card.(Example 8.21)

Solution:

$n(S)=52$

$(i) n(A)=26,\quad P(A)=\frac{26}{52}=\frac{1}{2}$

$(ii) n(B)=13,\quad P(B)=\frac{13}{52}=\frac{1}{4}$

$(iii) n(C)=2,\quad P(C)=\frac{2}{52}=\frac{1}{26}$

$(iv) n(D)=12,\quad P(D)=\frac{12}{52}=\frac{3}{13}$

$(v) n(E)=36,\quad P(E)=\frac{36}{52}=\frac{9}{13}$

FOR PRACTICE

21)The king and queen of diamonds , queen and jack of hearts ,jack and king of spades are removed from a deck of 52 playing cards and then well shuffled.Now one card is drawn at random from the remaining cards . Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card.
(Exercise8.3 (11))

Solution:

$n(S)=52-6=46$

$(i) n(A)=13,\quad P(A)=\frac{13}{46}$

$(ii) n(B)=0,\quad P(B)=0$

$(iii) n(C)=1,\quad P(C)=\frac{1}{46}$

22) From a well shuffled pack of 52 cards . a card is drawn at random .Find the Probability of it being either a red king or a black queen.(Exercise8.4 (7))

Solution:

$n(S)=52\quad n(A)=2\quad n(B)=2$

A *and* B are mutually exclusive events

$$P(A)=\frac{2}{52}, P(B)=\frac{2}{52}$$

$$P(A\cup B)=P(A)+P(B)$$
$$=\frac{2}{52}+\frac{2}{52}=\frac{4}{52}=\frac{1}{13}$$

23) A card is drawn from a pack of 52 cards . Find the probability of getting a king or heart or red card.
(Example 8.30)

Solution:

$n(S)=52$

$P(A)=\frac{4}{52}\quad P(B)=\frac{13}{52}\quad P(C)=\frac{26}{52}$

$P(A\cap B)=\frac{1}{52}\, P(B\cap C)=\frac{13}{52}\, P(C\cap A)=\frac{2}{52}$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

$$=\frac{4}{52}+\frac{13}{52}+\frac{26}{52}-\frac{1}{52}-\frac{13}{52}-\frac{2}{52}+\frac{1}{52}=\frac{28}{52}=\frac{7}{13}$$

FOR PRACTICE

24) If A,B,C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if.(Exercise 8.4 (13))

Solution:

$P(A\cap B)=\frac{1}{6}, P(B\cap C)=\frac{1}{4}, P(A\cap C)=\frac{1}{8},$

$P(A\cup B\cup C)=\frac{9}{10}, P(A\cap B\cap C)=\frac{1}{15}$

Then find P(A) , P(B) and P(C) .

25) Three fair coins are tossed together. Find the probability of getting
(i) all heads (ii) atleast one tail (iii) atmost one head (iv) atmost two tails.(Exercise 8.3 (8))

Solution:

$S=\{HHH.HHT,HTH,THH,HTT,THT,TTH,TTT\}$

$n(S)=8$

$(i)\, A=\{HHH\}\quad n(A)=1\quad P(A)=\frac{1}{8}$

$(ii)\, B=\{HHT,HTH,THH,HTT,THT,TTH,TTT\}$

$n(B)=7\qquad P(B)=\frac{7}{8}$

$(iii) C=\{HTT,THT,TTH,TTT\}$

$n(C)=4\qquad P(B)=\frac{4}{8}=\frac{1}{2}$

$(iv) D=\{HHH,HHT,HTH,THH,TTH,THT,HTT\}$

$n(D)=7\qquad P(D)=\frac{7}{8}$

26) In a game the entry fee is ₹150. The game consists of tossing a coin three times .Dhana bought a ticket for entry .If one or two heads show , she gets her entry fee back.If she throws 3 heads , She receives double the entry fees.Otherwise she will lose.Find the Probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.(Exercise 8.3 (14))

Solution:

$S=\{HHH.HHT,HTH,THH,HTT,THT,TTH,TTT\}$

$n(S)=8$

$(i) A=\{HHH\}\; n(A)=1$

$P(A)=\frac{1}{8}$

$(ii)\, B=\{HHT,HTH,THH,HTT,THT,TTH\}$

$n(B)=6\qquad P(B)=\frac{6}{8}=\frac{3}{4}$

$(iii) C=\{TTT\}$

$n(C)=1\; P(C)=\frac{1}{8}$

27) Three Unbiased coins are tossed once.Find the Probability of getting atmost 2 tails or atleast 2 heads.(Exercise 8.4 (9))

Solution:

$S=\{HHH.HHT,HTH,THH,HTT,THT,TTH,TTT\}$

$n(S)=8$

$A=\{HHH,HHT,HTH,THH,TTH,THT,HTT\}\; n(A)=3$

$P(A)=\frac{7}{8}$

$B=\{HHH,HHT,HTH,THH\}$

$n(B)=4\qquad P(B)=\frac{4}{8}$

$A\cap B=\{HHH,HHT,HTH,THH\}$

$n(A\cap B)=4\qquad P(A\cap B)=\frac{4}{8}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{7}{8}+\frac{4}{8}-\frac{4}{8}=\frac{7}{8}$

28) A coin is tossed thrice . Find the Probability of getting exactly two heads or atleast one tail or two concecutive heads.(Exercise 8.4 (12))

Solution:

$S=\{HHH.HHT,HTH,THH,HTT,THT,TTH,TTT\}$

$n(S)=8$

$A=\{HHT,HTH,THH\}\; n(A)=3\quad P(A)=\frac{3}{8}$

$B=\{HHH,HHT,HTH,THH,TTH,THT,HTT,TTT\}$

$n(B)=7\qquad P(B)=\frac{7}{8}$

$C=\{HHT,THH,HHH\}$

$n(C)=3\qquad P(C)=\frac{3}{8}$

$A\cap B=\{HHT,HTH,THH\}$

$n(A\cap B)=3\qquad P(A\cap B)=\frac{3}{8}$

$B\cap C=\{HHT,THH\}$

$n(B\cap C)=2 \quad P(B\cap C)=\frac{2}{8}$

$C\cap A=\{HHT,THH\}$

$n(B\cap C)=2 \quad P(B\cap C)=\frac{2}{8}$

$A\cap B\cap C=\{HHT,THH\}$

$n(A\cap B\cap C)=2 \quad P(A\cap B\cap C)=\frac{2}{8}$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

$$=\frac{3}{8}+\frac{7}{8}+\frac{3}{8}-\frac{3}{8}-\frac{2}{8}-\frac{2}{8}+\frac{2}{8}=\frac{8}{8}=1$$

29)A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls.One ball is drawn at random from the bag .Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black.(Exercise 8.3 (9))

Solution:

$n(S)=5+6+7+8=26$

$(i) n(A)=6$

$P(A)=\frac{n(A)}{n(S)}=\frac{6}{26}=\frac{3}{13}$

$(ii). n(B)=8+5=13$

$P(B)=\frac{n(B)}{n(S)}=\frac{13}{26}=\frac{1}{2}$

$(iii). n(C)=26-6=20$

$P(C)=\frac{n(C)}{n(S)}=\frac{20}{26}=\frac{10}{13}$

$(iv). n(D)=5+7=12$

$P(D)=\frac{n(D)}{n(S)}=\frac{12}{26}=\frac{6}{13}$

FOR PRACTICE

30)A bag contains 6 green balls, Some black and red balls .Number of black balls is as twice as the number of red balls.Probability of getting a green ball is thrice the probability of getting a red ball.Find (i) number of black balls (ii) total number of balls.(Example 8.24)

31) A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i). then find x.(Exercise 8.3(6))

Solution:

$(i) \quad n(S)=12+x$

$n(R)=x$

$P(R)=\frac{x}{12+x}$

$(ii) n(S)=20+x$

$P(R_1)=\frac{x+8}{20+x}$

$P(R_1)=2P(R)$

$\frac{x+8}{20+x}=2\left(\frac{x}{12+x}\right)$

$(x+8)(x+12)=2x(20+x)$

$x^2+20x+96=40x+x^2$

$2x^2+40x-x^2-20x-96=0$

$(x+24)(x-4)=0$

$x=-24 \;,\; x=4$

$\Rightarrow x=4$

$(i) P(R)=\frac{4}{16}=\frac{1}{4}$

32) A box contains cards numbered 3,5,7,9,……….35,37. A card is drawn at random from the box.Find the probability that the drawn Card have either multiples of 7 or a prime number. (Exercise 8.4 (8))

Solution:

$n(S)=18$

$A=\{7,21,35\} \quad n(A)=3 \quad P(A)=\frac{3}{18}$

$B=\{3,5,7,11,13,17,19,23,29,31,37\}$

$n(B)=11 \quad P(B)=\frac{11}{18}$

$A\cap B=\{7\} \quad n(A\cap B)=1$

$P(A\cap B)=\frac{1}{8}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{3}{18}+\frac{11}{18}-\frac{1}{18}=\frac{13}{18}$

33) In a class of 50 students , 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS.One of the students is selected at random. Find the Probability that (i) The student opted for NCC but not NSS (ii) The student opted for NSS but not NCC (iii)The student opted for exactly one of them. (Example 8.31)

Solution:

$n(S)=50 \quad n(A)=28 \quad n(B)=30 \quad n(A\cap B)=18$

$P(A)=\frac{n(A)}{n(S)}=\frac{28}{50} \qquad P(B)=\frac{n(B)}{n(S)}=\frac{30}{50}$

$P(A\cap B)=\frac{18}{50}$

$(i)P(A\cap\bar{B})=P(A)-P(A\cap B)=\frac{28}{50}-\frac{18}{50}=\frac{1}{5}$

$(ii)P(\bar{A}\cap B)=P(B)-P(A\cap B)=\frac{30}{50}-\frac{18}{50}=\frac{6}{25}$

$(iii)P(A\cap\bar{B})\cup P(\bar{A}\cap B)=\frac{11}{25}$

FOR PRACTICE

34) A and B are two candidates seeking admission to IIT.The Probability that A getting selected is 0.5 and the Probability that both A and B getting selected is 0.3. Prove that the Probability of B being selected is atmost 0.8.(<u> Example 8.32)</u>

Solution:

$P(A)=0.5, P(A\cap B)=0.3$

$P(A\cup B)\le 1$

$P(A)+P(B)-P(A\cap B)\le 1$

$0.5+P(B)-0.3\le 1$

$P(B)\le 1-0.2$

$P(B)\le 0.8$

35) If A and B are two events such that

$P(A)=\frac{1}{4} \quad P(B)=\frac{1}{2}$ P(A *and* B) = $\frac{1}{8}$ எனில்

(i) P(A *or* B) = $\frac{1}{8}$ (ii) P(not A and not B).

(<u> Example 8.29)</u>

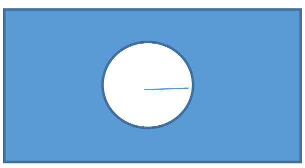
36) The Probability of happening an event A is 0.5 and that of B is 0.3 .if A and B are mutually exclusive events, then find the Probability that neither A nor B happen.(<u> Exercise 8.4 (5))</u>

Solution:

$P(AUB)=P(A)+P(B)=0.5+0.3=0.8$

$P(\bar{A}\cap\bar{B})=P(\overline{AUB})=1-P(AUB)=1-0.8=0.2$

37).Some boys are playing a game in which the stone thrown by them by landing in a circular region. (<u> Exercise 8.3 (12))</u>



Solution:

Area of the rectangle $=l\times b=3\times 4=12$

$n(S)=12$

$n(A)=\pi r^2=3.14\times 1^2=3.14$

$P(A)=\frac{n(A)}{n(S)}=\frac{3.14}{12}=\frac{314}{1200}=\frac{157}{600}$

38).In a box there are 20 non-defective bulbs . if the Probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then find the number of defective bulbs.(<u> Exercise 8.3 (10))</u>

Solution:

Number of defective bulbs=x

number of non defective bulbs =20

$n(S)=20+x$

$p(A)=\frac{3}{8}$

$\frac{n(A)}{n(S)}=\frac{3}{8}$

$\frac{x}{20+x}=\frac{3}{8}$

$8x=60+3x$

$5x=60$

$x=12$